Optics I

University of Baghdad
College of Science
Department of Physics

Subject: Optics I
Semester: First
Year: First Year Physics

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References:
Halliday, Resnick and Walker; Fundamentals of Physics; 8th edition 2008.
F.Sears, Addison-Wesley publishing company , Optics 1964.
F.Jenkins& H.White, Fudamentals of Optics by , McGraw Hill book company,4th edition ,1985.

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Dr. Sarmed S Alawadi

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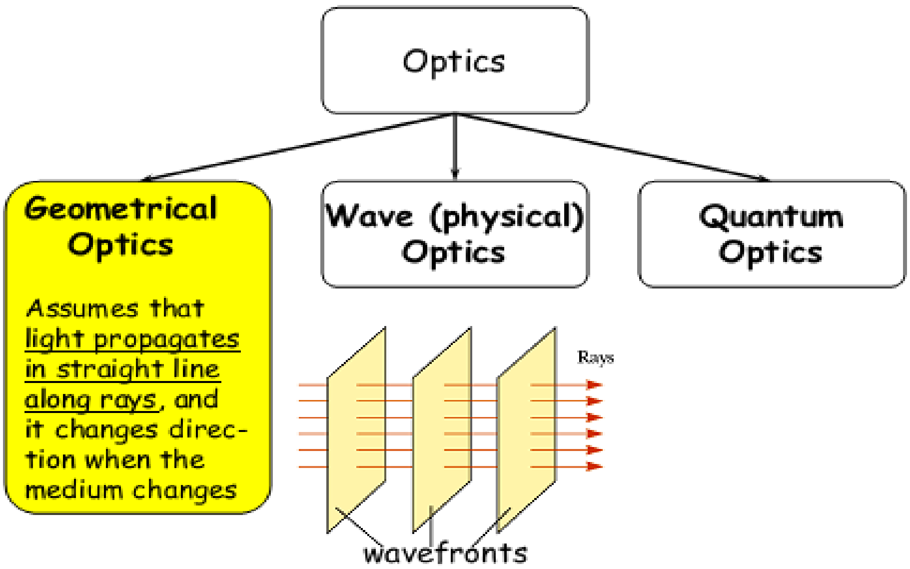
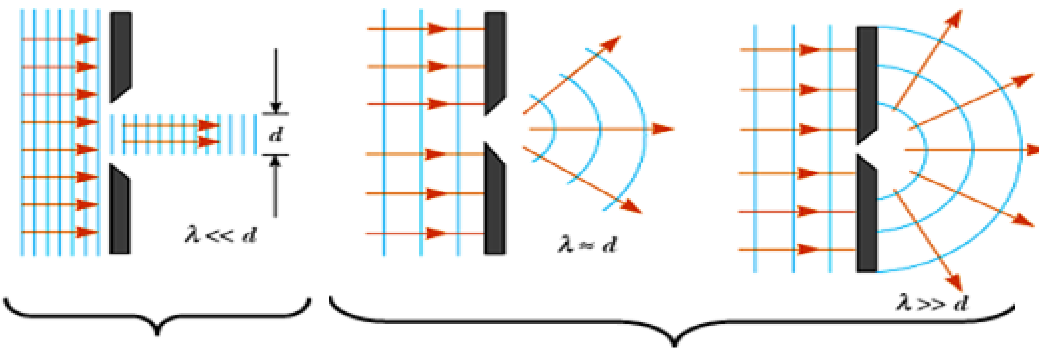
Chapter One

Nature and propagation of light

- **Optics-----** *is the branch of physics which involves the behavior and properties of light, including its interactions with matter and the construction of instruments that use or detect it.*

- **Geometrical Optics----***Subset of optics concerning interaction of light with macroscopic material*
  Dimension larger than a human hair ≈ 50 mm





geometric optics is a good approximation whenever light's wavelength is negligibly small compared to other relevant length scales

geometric optics breaks down when light's wavelength cannot be neglected (e.g., diffraction occurs when the size of the slit is ≤ λ)

- **So Light can travel through:**
- empty space,
- air, glass, water,
- cornea, eye lens etc.

  Each one referred to as a medium

A Brief History of Light

- From around the 6th-5th century BC
  - In ancient India, the Hindu schools developed theories on light
- Al-Hasan Ibn AI-Haitham
  - Put it the basics of optics and wrote a lot of in it. He's books which authored in 11th century in optics science still as references until the 17th century
- 1000 AD
  - It was proposed that light consisted of tiny particles
- Newton
  - Used this particle model to explain reflection and refraction
- Huygens
  - 1678
  - Explained many properties of light by proposing light was emitted in all directions as a series of waves in a medium
- Young
  - 1801
  - Strong support for wave theory by showing interference
- Maxwell
  - 1873
  - Showed that an oscillating electrical circuit should radiate electromagnetic waves. The velocity of propagation of the waves could be reflected, refracted, focused by a lens, polarized, and so on, just as could waves of light
- Planck
  - EM radiation is quantized
    - Implies particles
  - Explained light spectrum emitted by hot objects
- Einstein
  - 1900
  - Wave-Particle nature of light (dual nature)
  - Explained the photoelectric effect

The ***modern concept of light contains elements of both Newton's and Huygens descriptions***. Light is said to have **"dual nature"**; certain phenomena, such as **interference**, exhibit the wave nature of light. Other phenomena" the photoelectric effect, for example, display the particle aspect of light.

- The Particle Nature of Light

  "Particles" of light are called *photons*

  Each photon has a particular energy

  $E = h\,f$

  h is *Planck's constant* = 6.63×10^{-34} J s

- Geometric Optics – Using a Ray Approximation
  - Light travels in a straight-line path in a homogeneous medium until it encounters a boundary between two different media
  - The *ray approximation* is used to represent beams of light

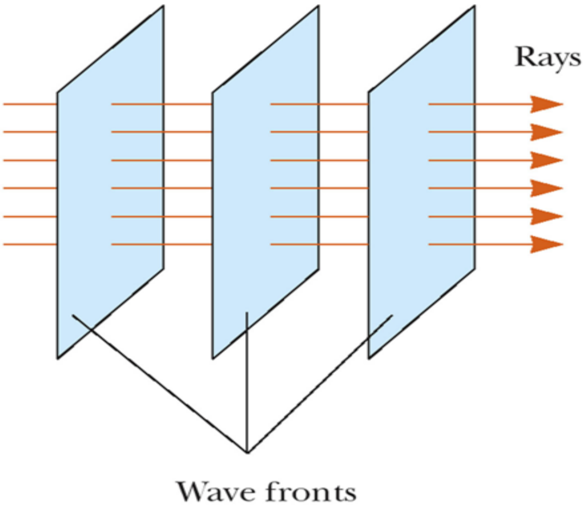
Example:

What is meant by wave front and rays?

Answer:

- A *wave front* is a surface passing through points of a wave that have the same phase and amplitude

- A *ray* of light is an imaginary line drawn along the direction of travel of the light beams. The rays, corresponding to the direction of the wave motion, are perpendicular to the wave fronts



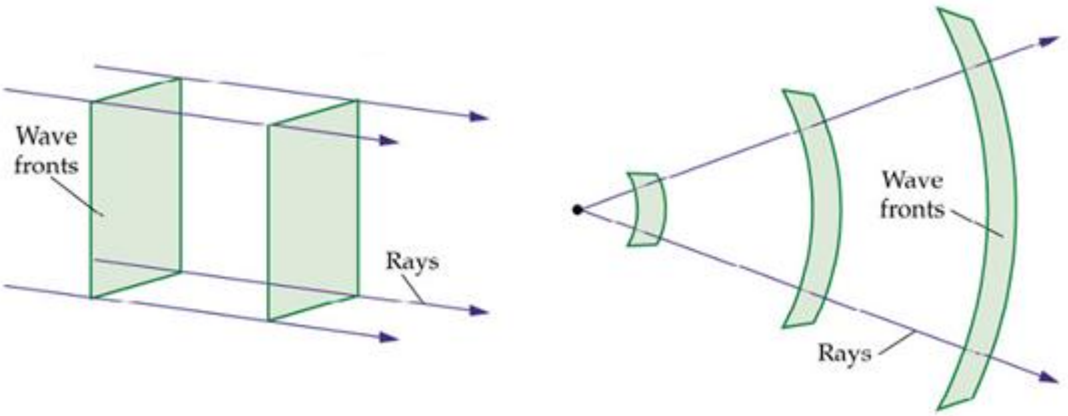
So we must understand that:

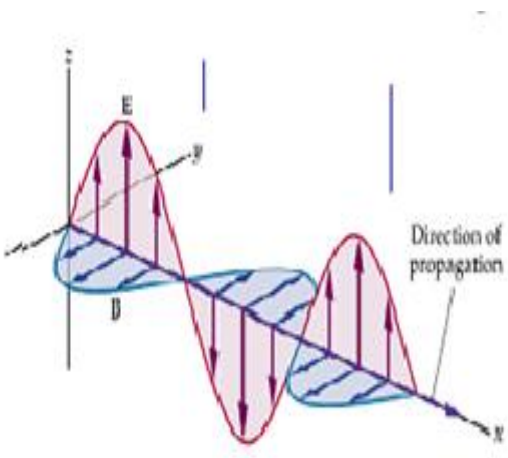
- In describing the propagation of light as a wave we need to understand two ideas:

1. wave fronts: a surface passing through points of a wave that have the same phase.
   - We can choose to associate the wave fronts with the instantaneous surfaces where the wave is at its maximum.
   - Wave fronts travel outward from the source at the speed of light: c.
   - Wave fronts propagate perpendicular to the local wave front surface.

2. rays: a ray describes the direction of wave propagation.

   - The propagation of the wave fronts can be described by light rays when a small section near the optical axis is considered.
   - And in free space, the light rays can be thought of as they travel in straight lines, perpendicular to the Wave fronts.



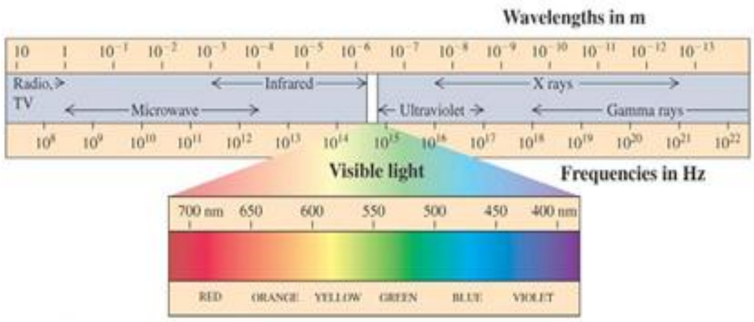


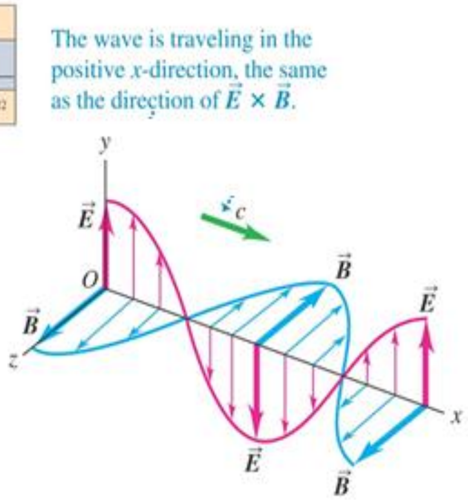
- **What is light?**
  - Light is a form of electromagnetic energy –detected through its effects, e.g. heating of illuminated objects, conversion of light to current, mechanical pressure (“Maxwell force”) etc.
  - Light energy is conveyed through particles: “photons”–particle behavior, e.g. shadows
  - Light energy is conveyed through waves–wave behavior, e.g. interference, diffraction
  - Quantum mechanics reconciles the two points of view, through the “wave-particle duality” assertion

Electromagnetic radiation is a type of energy consisting of oscillating electric (E) and magnetic (B) fields, which move through space.

- Electromagnetic radiation include:

Types of radiation		Frequency	Wavelength
"Wave" region	*radio waves*	10^9 Hz and less	300 mm and longer
	microwaves	10^9 Hz to 10^{12} Hz	300 mm to 0.3 mm
"Optical" region	*inf rared*	10^9 Hz to 4.3×10^{14} Hz	300 μ to 0.7 μ
	visible	4.3×10^{14} Hz to 5.7×10^{14} Hz	0.7 μ to 0.4 μ
	ultraviolet	5.7×10^{14} Hz to 10^{16} Hz	0.4 μ to 0.03 μ
"Ray" region	*X – rays*	10^{16} Hz to 101^9 Hz	300 A° to 0.3 A°
	gamma rays	10^{19} Hz and above	0.3 A° and shorter





Light is propagating electromagnetic waves

Nature and propagation of light

1. Foundation of optical phenomena are Maxwell's Equations
2. Light is an electromagnetic wave (electromagnetic spectrum)
3. Can also be viewed as a stream of corpuscles (particles)
4. Speed of light in vacuum is about 300,000Km/sec= 3 x 10^8 m/sec
5. Speed of light different in different materials ,but frequency remins constant according to :

Speed = frequency x wavelength = $v_{medium} = f \cdot \lambda_{medium}$ *(recall that $f=1/T$)*

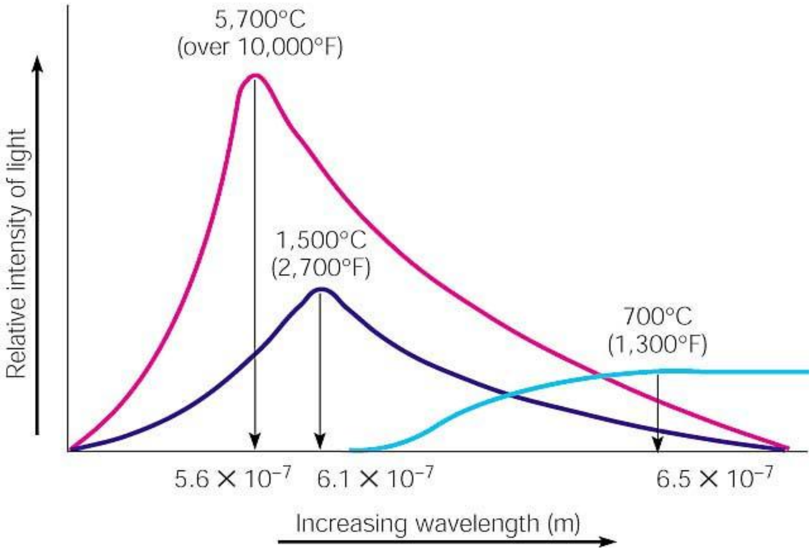
6. Visible light: 400 nm-700 nm(1nm =10^{-9} m)

1. **Source of light**
   1. **Microscopic level:**
   2. **Phenomenological level:**

- **Incandescent light (hot bulb , thermal excitation):**
  - When light is given off as a result of high temperatures an object is said to be incandescent.
  - The electromagnetic wave model is used to explain incandescent light.
- **Luminescent light (cool light) : fluorescent, chemi-luminescent, bio- luminescent (fireflies),tribo- luminescent (mechanical)[Luminous:** When something produces light it is said to be luminous]

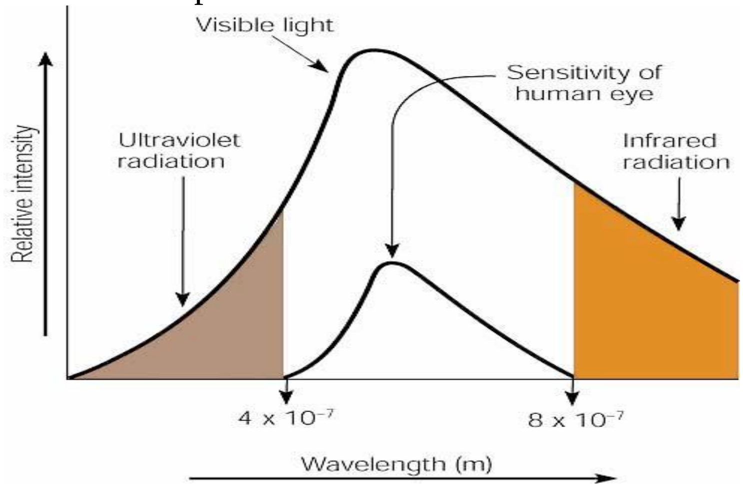
- **Black Body Radiation**

- Three different objects emitting blackbody radiation at three different temperatures. The intensity of blackbody radiation increases with increasing temperature and the peak wavelength emitted shifts toward shorter wavelengths.



- **Sun light**

  Sunlight is about 9 percent ultraviolet radiation, 40 percent visible light, and 51 percent infrared radiation before it travels through the earth's atmosphere



- **Course divided into two "halves".**

- Geometric Optics: based upon Newton’s idea. Propagation of light is governed by Fermat’s Principle.
- Wave (Physical) Optics: based upon Huygens’ idea. Propagation of light is governed by Huygens' Principle (wave motion).

In other word

- **Optics – two approaches**
  1. **Ray tracing (geometrical optics)**
  2. **Wave optics (wave properties fundamentally important)**

- **Waves and geometrical optics**
  1. **geometrical optics**↔ wavelength<< objects under study
  2. **ray** = line giving the direction of travel of wave; it is perpendicular to the wave fronts (surface constant phase)
  3. **media boundaries:** incident; reflected; refracted rays
  4. **index of refraction:**

$n = c/v_{medium} \geq 1$ = speed of light in vacuum / speed of light in medium ≥ 1

thus $\quad v_{medium} = c/n$

$$n = \frac{c}{v} = \sqrt{\frac{\mu\,\varepsilon}{\mu_o \varepsilon_o}} = \sqrt{\mu_r \varepsilon_r}$$

$$\mu_r = \frac{\mu}{\mu_o} = \text{relative permittivity} = \text{dielectric constant.}$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o} = \text{relative permeability.}$$

For non- magnetic material, μ_r=**1**.

$$n = \sqrt{\varepsilon_r}$$

5. **light is slower in matter**:

Material	**Index (*n*)**	***v_{medium}***
• Vacuum	1.00	300000 Km/s
• Air	1.0003	
• Water	1.333	225564 Km/s
• Glass	1.5(depend on type)	
• Diomond	2.42	123967 km/s

6. ***n is one of important properties of materials*** ;

n depend on λ → n = n(λ)(chromatic dispersion), which mean that the variation of the index of refraction with frequency cause chromatic dispersion.

- **Index of Refraction n:**

□ In materials, light interacts with atoms/molecules and travels slower than it can in vacuum, e.g.,

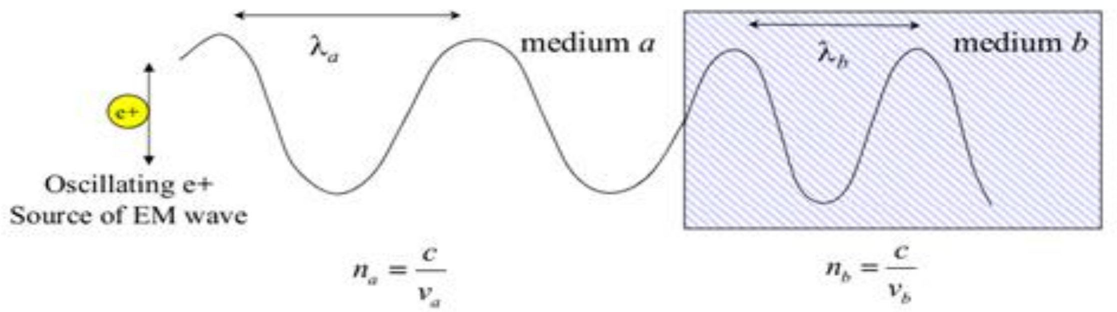
$$v_{water} \cong \frac{3}{4}c$$

□ The optical property of transparent materials is called the Index of Refraction:

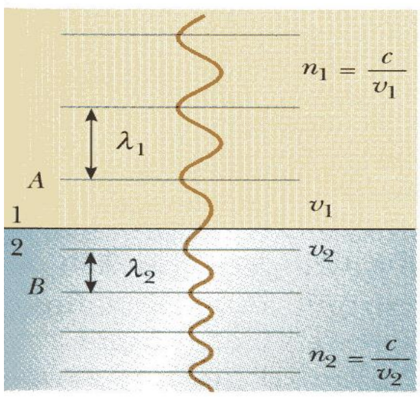
$$n \equiv \frac{c}{v_{material}}$$

□ Since $v_{\text{material}} < c$ always $n > 1$!

- **Index of Refraction and Wave Aspects of light:**



Note: frequency of the EM wave is dictated by the oscillations of the charge and the timing of this oscillation can't change for an observer in either medium a or b.



f does not change across media

Recall for a EM wave, we have: $v_a = f\lambda_a$ *and* $v_b = f\lambda_b$

So, in the two medium, we have $v_a = f\lambda_a$ *and* $v_b = f\lambda_b$

Dividing these two equations, we have:

$$\frac{va}{vb} = \frac{f\,\lambda a}{f\,\lambda b} = \frac{\lambda a}{\lambda b} \rightarrow \frac{\lambda a}{\lambda b} = \frac{c/na}{c/nb} \rightarrow \frac{\lambda a}{\lambda b} = \frac{nb}{na}$$

So, the wavelength of a light must change in different medium accordingly,

$$n_a\,\lambda_a = n_b\,\lambda_b$$

With one medium being a vacuum, we have $\lambda_n = \lambda / n$

Examples

1-Light – Wave or stream of particles?
Answer: Yes! As we'll see below, there is experimental evidence for both interpretations, although they seem contradictory.

2-What is a wave?
Answer: More familiar types of waves are sound, or waves on a surface of water. In both cases, there is a perturbation with a periodic spatial pattern which propagates, or travels in space. In the case of sound waves in air for example, the perturbed quantity is the pressure, which oscillates about the mean atmospheric pressure. In the case of waves on a water surface, the perturbed quantity is simply the height of the surface, which oscillates about its stationary level

There are five primary properties of light are:

1. Intensity,
2. Frequency or Wavelength,
3. Polarization,
4. Phase,
5. Orbital angular momentum.

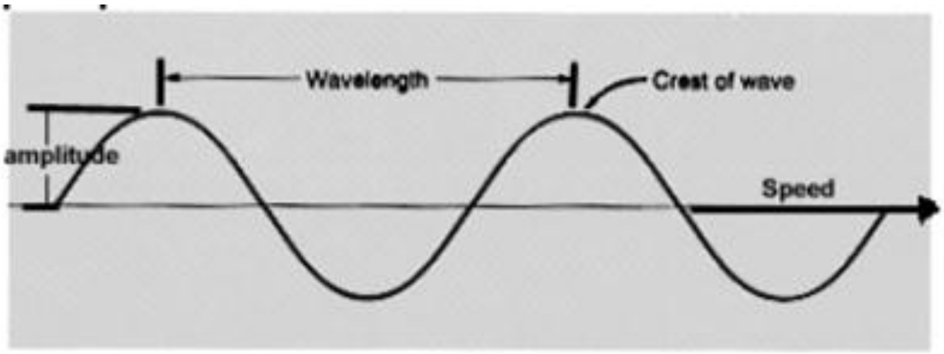
- **Properties of Waves**
  - A wave is described by four properties

    –The wavelength (λ) - units of length
    –The amplitude of the wave (a)
    –The speed of the wave (c) -units of speed length/time

–The frequency of the wave (f) - units of 1/time

- Three of these properties are interrelated

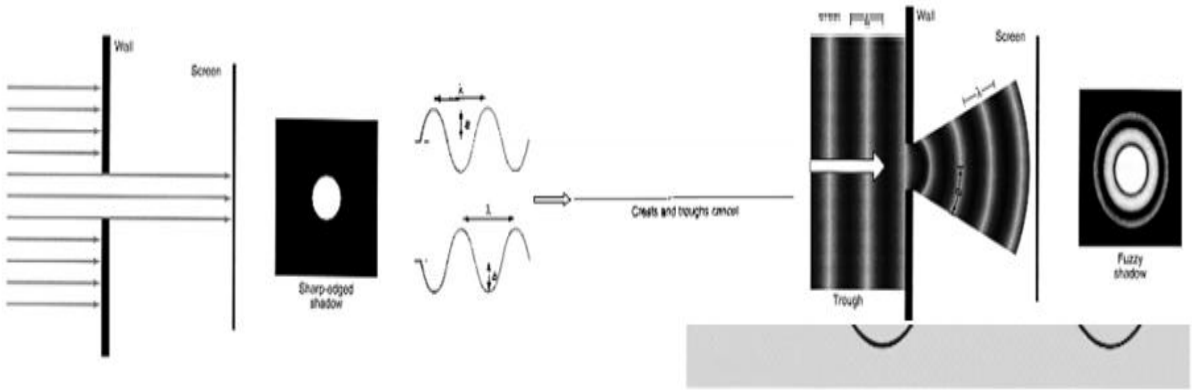
–$c = \lambda \times f$



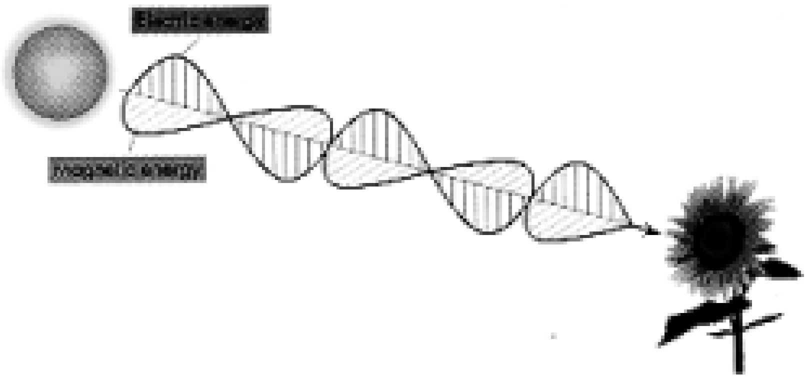
- **The Wave Nature of Electro-Magnetic Radiation**
    - Light is just one form of electro-magnetic radiation
    - Light often behaves like a wave;

–Diffraction

–Interference



- James Clark Maxwell (1831-1879) derived 4 laws which describe electricity and magnetism
    - These are known as <u>Maxwell's equations</u>
    - These predict that a varying magnetic field causes an electric field and vice versa
    - Maxwell's equations correctly predict the speed of light
- All EM radiation is composed of two waves
    - An electric field
    - A magnetic field
    - The fields are orthogonal
    - It needs no medium to be transmitted through
    - Energy is carried by the wave



- Maxwell equations is:

$$\nabla \bullet \vec{E} = 0 \qquad (1.1a)$$

$$\nabla \bullet \vec{B} = 0 \qquad (1.2a)$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \qquad (1.3a)$$

$$\nabla \times \vec{B} = +\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \qquad (1.4a)$$

The properties of the light according to Maxwell, are:

1. Have a wave nature.
2. It is a transverse wave.
3. Speed of light is $3x10^8$ m/sec.
4. There is a relation between the relative values for the electric and magnetic fields.
5. If an electric and magnetic field effect on a certain point, energy flux will happen, i.e. have momentum for the same point.

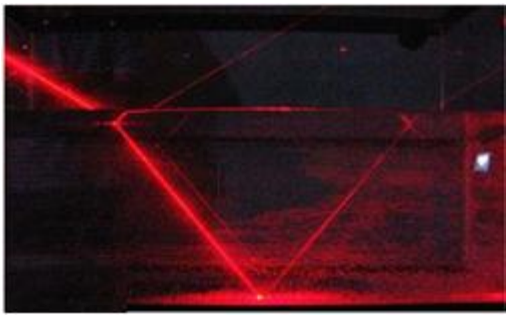
Example:

Today I noticed a maple tree outside of my office window had turned orange. It looks orange to me because it is reflecting orange light (λ=670 nm) while absorbing the other colors of visible light. What is the energy of a photon of orange light?

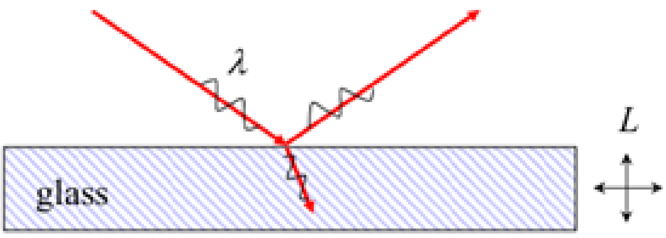
The Study of Light: Optics

- Geometric (or Rays) Optics:
  - ❖ In most daily situations, light (rays) travel in a straight line in a uniform medium.
  - ❖ At the boundary between two materials (air & glass), a ray's direction might change.
  - ❖ Wave characteristics of light are not important.

Geometric Optics is the study of the propagation of light with the assumption that rays are straight lines in a fixed direction through an uniform medium.



- **Condition for Rays Optics:**



L >> λ

Relevant system size >> wavelength;
In this approximation, wave characteristic of light is not important and rays model of light gives accurate predictions.
(Visible light: λ ~ 500 nm << L→Rays Optics works well with typical optical instruments: mirror, lens, cameras, telescopes,…)

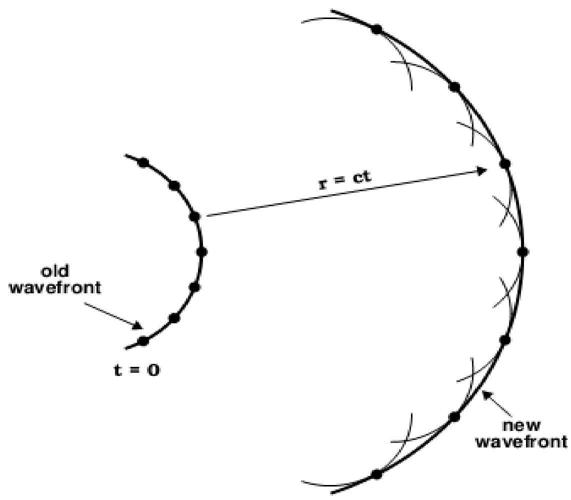
Huygens’ Principle:

- 1. In 1678, Huygens proposed the wave theory of light.

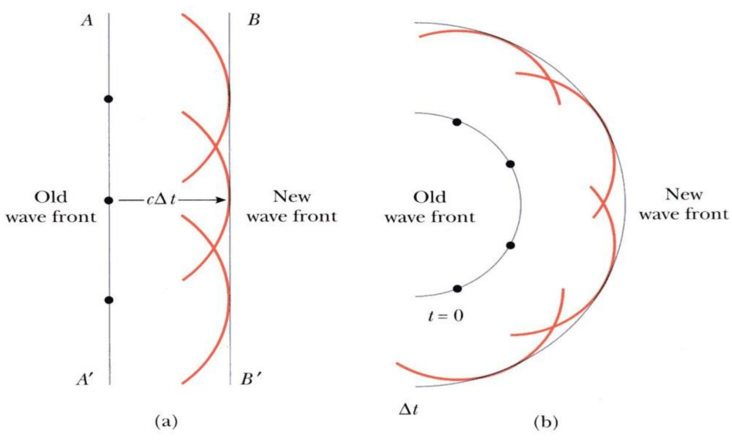
At that same time, Newton maintained that light acted like a particle. As we saw in the last section, Planck derived in 1900 that light has the characterizes of both a wave and a particle = ⇒ a wavelet, which he called a **photon.**

- 2. Huygens state that *"every point of a wave front may be considered as the source of small secondary wavelets, which spread out in all directions from their centers with a velocity equal to the velocity of propagation of the wave"*.

  After time t, the new wave front is then found by constructing a surface tangent to the secondary wavelets, or as it is called, the "envelope" of the wavelets. Huygens' Principle is illustrated in figures bellow.



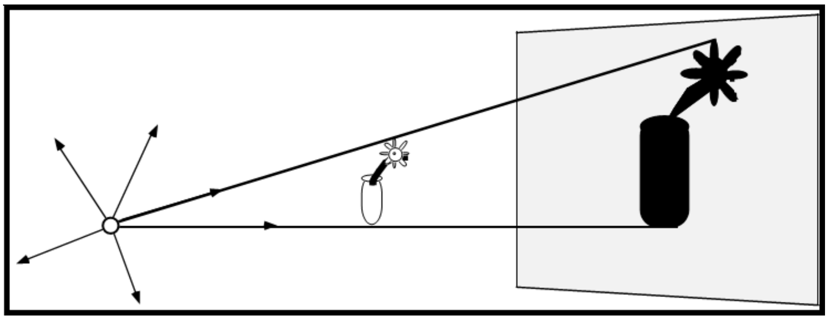
i.e: All points on a wave front act as new sources for the production of spherical secondary waves



- **Shadows**

  A **shadow** *"is an area where direct light from a light source cannot reach due to obstruction by an object, it occupies all of the space behind an* ***opaque*** *object with light in front of it.*

We suppose that when light travels through a homogeneous medium it moves along straight lines. That observation is often called the law of rectilinear propagation. The existence of shadows is good evidence for the ray model of light. <u>When light from a small (or 'point') source goes past the edges of an opaque object it keeps going in a straight line, leaving the space behind the object dark</u>



The sharpest shadows are from -

a) Large bulbs like an overhead projector
b) Small bulbs
c) Candles
d) Two light sources
e) Fluorescent tubes

i.e <u>The best shadows come from a point source, for example a small light source or candle</u>

• To figure out the shadow location, we use geometrical optics - light travels in straight lines.

<u>Example:</u>

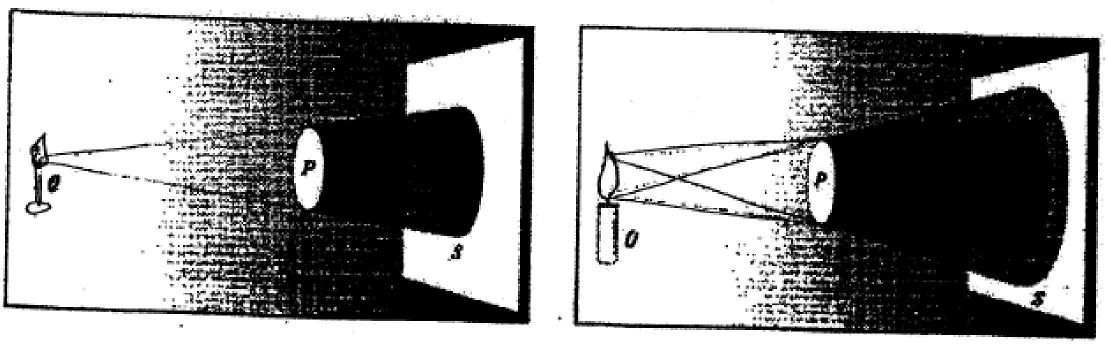
Which light source will give a sharper shadow, the Fluorescent tubes or the lamp?

Shadows tell us -

a) What direction the light is shining from
b) That something is blocking the light
c) That light travels in straight lines
d) Where energy is being absorbed

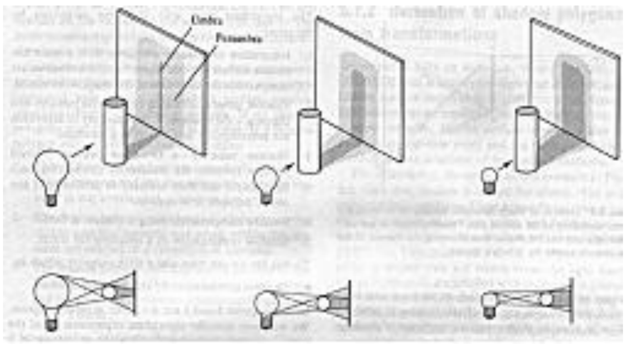
- <u>Shadow Generation -*Point Light Source*</u>
- Hard shadows
- Sharp transition from dark shadow to brightly-lit non-shadow region
- For a point source there is one shadow region called **umbra**

- <u>Shadow Generation - Real Light Sources</u>
- Generally have measure
- able cross-section
- Two shadow regions
  1. **Umbra** : is the region behind the obstacle that receives no light at all
  2. **Penumbra**: is the region that receives partial light from the source and surrounded the umbra region.
     - Its large when the source is extensive (limit is ambient!) or Source is close
     - Its small when the Source is small or ar away



shadow of (a) point source, (b) not a point source

- Umbra/Penumbra ratio==decreases
  - with increasing size of source,
  - with distance

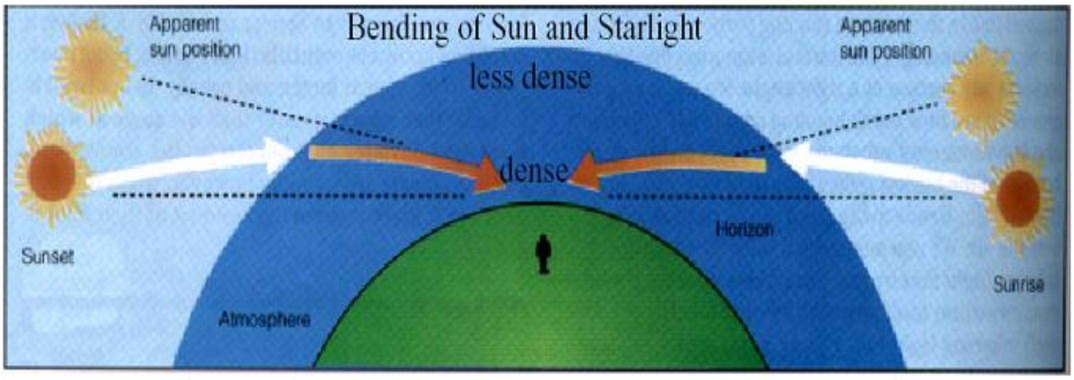


So we can compute the length of the shadow <u>when</u> we know the length of the object (a) and the angle between the point of light and the object (θ), (i.e.):

Length of shadow = $\tan(\theta) \times a$

Atmospheric Refraction:

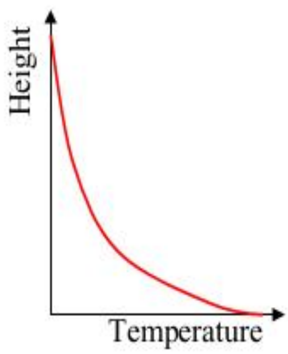
- The velocity of light in all material substances is less than its velocity in free space and in a gas the velocity decreases as the density increases. The density of the earth's atmosphere is greatest at the surface of the earth and decreases with increasing elevation. As a result, light waves entering the earth's atmosphere are continuously deviated in below fig.



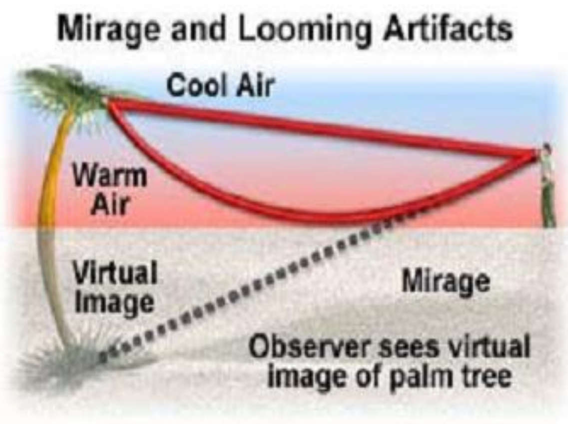
The bending of the sun's rays by the atmosphere causes the sun to rise about two minutes earlier and set about two minutes later than it otherwise would.

- Another phenomenon produced by atmospheric refraction is the *mirage*. A mirage is an optical effect of the atmosphere caused by refraction when light passes from air with one density into air with a different density and the object appears displaced from its true position

- The deviation of light by atmospheric refraction decreases with increasing angle of elevation of the light above horizontal, falling to zero for light incident normally on the earth's surface.

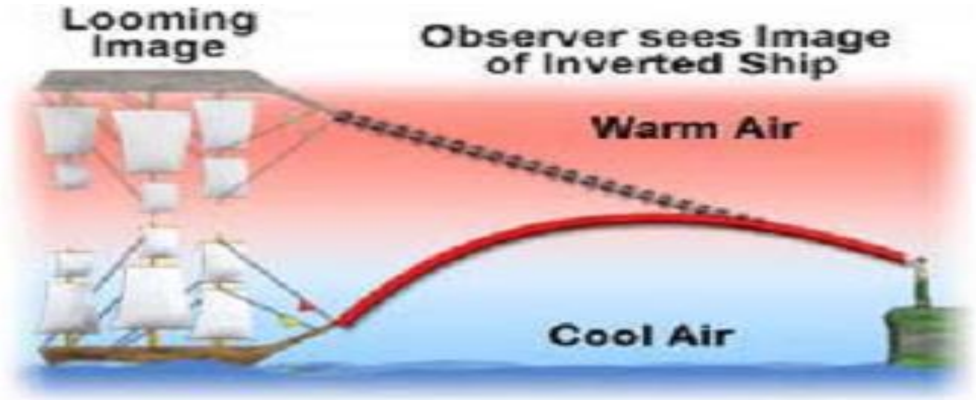


Temperature profile above sand



- -The mirage called an *inferior mirage* occurs when the image appears below the true location of the observed object.

- - During a phenomenon called *looming*, objects sometimes appear to be suspended above the horizon.
  - Looming is considered a *superior mirage* because the image is seen above its true position.



- A mirage that changes the apparent size of an object is called towering.
- A type of towering, called *Fata Morgana*, is frequently observed in coastal areas as towering castles that appear out of thin air.

Fermat's Principle:

- The laws of reflection and refraction may be show to follow from a general principle first stated by Fermat in 1658.

- In optics, Fermat's principle or **the principle of least time** is ***the principle that the path taken between two points by a ray of tight is the path that can be traversed in the least time***. This principle is sometimes taken as the definition of a ray of light.

- Therefore the mathematical formula for Fermat principle is $\frac{d\,(OPL)}{dx} = 0$

- It can be used to describe the properties of light rays ***reflected*** off mirrors, ***refracted*** through different media, or undergoing total internal reflection.

- The path taken by a ray of light between two points is path of shortest time, but

  $$\boldsymbol{t = l\,/\,c}$$

  where *l* is the optical path length, or time (t) is equal to the distance traveled *l* at a particular velocity (v).

  $$\boldsymbol{t = l\,/\,v}$$

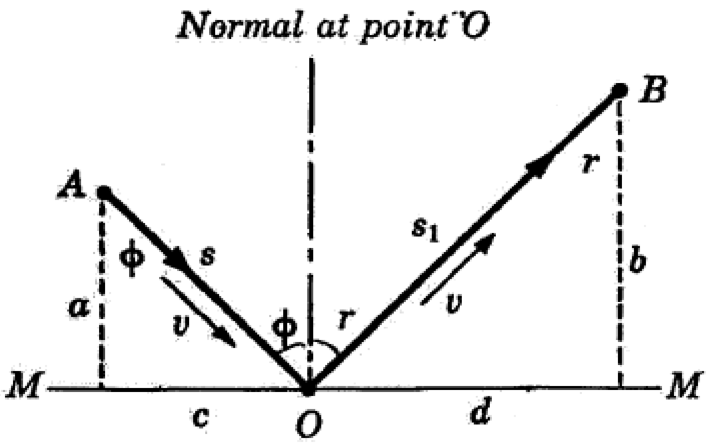
- The mathematical formula for Fermat's principle for the ray travels from an arbitrary point **"P"** to another arbitrary point **"Q"** is:

$$\boldsymbol{t = l_1\,/\,v_1 + l_2\,/\,v_2}$$

- **A derivation of "Snell's Law of Reflection"**

If υ represents the velocity of propagation. The length of path is $s+s_1$, and the time t along the path is:

$$t = \frac{s + s_1}{v}$$



It is seen from the diagram that:

$$s = a \sec \phi$$

$$s_1 = b \sec r$$

It follows from the two preceding equation that:

$$t = \frac{1}{v}(a \sec \phi + b \sec r)$$

If the point O displaced slightly, the angles ϕ and r will change by d ϕ and dr, and the corresponding change dt in the time is:

$$dt = \frac{1}{v}(a \sec \phi \tan \phi \, d \phi + b \sec r \tan r \, dr)$$

If the time is a minimum, dt = 0 and

$$a \sec \phi \tan \phi \, d \phi = - b \sec r \tan r \, dr$$

The differential dϕ and dr are not independent. From the figure:

$$c + d = constant = a \tan \phi + b \tan r$$

Taking differentials of both sides, we obtain:

$$0 = a \sec^2 \phi \, d \phi + b \sec^2 r \, dr$$

When the condition for minimum time is combined with this equation, we obtain:

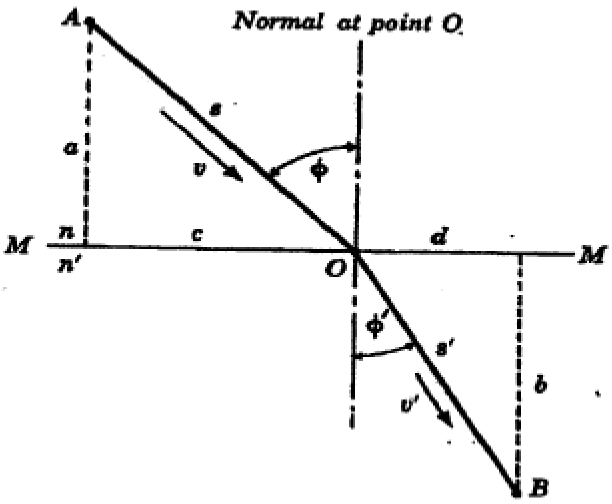
$$\sin \phi = \sin r$$

$$\phi = r$$

- That is the light path AOB that is traversed in the shortest time is that for which the angle of reflection equals the angle of incidence.

- **A derivation of "Snell's Law of Refraction"**

In figure below, MM represents the boundary plane between two substances having indices of reflection n and n', and velocities υ and υ'. AOB is a path of a ray from A to B, ϕ and ϕ' are the angles of incidence and of refraction. The time from A to B is:



$$t=\frac{s}{v}+\frac{s'}{v'}=\frac{a\sec\phi}{v}+\frac{b\sec\phi'}{v'}$$

If the point O is displaced slightly,

$$dt=\frac{a\ \sec\phi\ \tan\phi\ d\varphi}{v}+\frac{b\ \sec\phi'\ \tan\phi'\ d\phi'}{v'}$$

If the time minimum, dt = 0 and

$$\frac{a\ \sec\phi\ \tan\phi\ d\phi}{v}=-\frac{b\ \sec\phi'\ \tan\phi'\ d\phi'}{v'}$$

Also, since c + d= constant,

$$a\sec^2\phi\ d\phi=-b\sec^2\phi'\,d\phi'$$

Dividing one of the preceding equations by the other, obtain:

$$\frac{\sin\phi}{v} = \frac{\sin\phi'}{v'}$$

Since υ=c/n and υ'=c/n', this reduces to:

$$n \sin \phi = n' \sin \phi'$$

- **Optical Path Length and Refractive Index:**

Optical Path Length (OPL) *is the distance light travels in a vacuum (also air) in the same time it travels a distance L in a medium.*

In free space light travels in straight lines. For two points separated by a distance L_{air} time taken for the light to travel between them is

$$t = L_{air} / c$$

where c is the speed of light, being 2.998×10^8 m/sec

If there is a media between the two points (say glass), the speed of light is reduced by a factor n, so the time is:

$$t_n = n\, L_{air} / c$$

Where n is the refractive index a physical characteristic of the medium.

Useful to introduce the optical path length l, physical path length scaled by the refractive index, so

$$l = n\, L_{air} \quad \Rightarrow \quad t_n = l / c$$

The refractive index for a material is not constant, dependence known as dispersion, depends on:

Gases: pressure and wavelength.

Liquids: temperature and wavelength

Solids: wavelength, and slightly on temperature.

Glasses: wavelength, slightly on temperature.

Note: refractive index of air ≈ 1, normally taken at exactly unit

Because light always travels slower in a material (medium), the optical path length (distance in air) will always be longer than the actual thickness *l* of the medium.

Distance = velocity · time

$$\mathbf{L_{air} = c.\, t}$$

$$\boldsymbol{l} \mathbf{= v.\, t = (c / n).\, t}$$

$$\therefore \quad \mathbf{L_{air} = OPL = n.\, \boldsymbol{l}}$$

If light travels through "m" different media, the total OPL is the sum of the optical paths in each of the different media:

$$\mathbf{OPL_{total} = n_1.\boldsymbol{l}_1 + n_2.\, \boldsymbol{l}_2 + \ldots\ldots + n_m.\, \boldsymbol{l}_m}$$

$$\mathbf{OPL_{total}} = \sum_{i=1}^{m} \mathbf{ni.\, \boldsymbol{l}i}$$

Optical Path Difference:

Optical Path Difference (OPD) *is the difference between two optical path lengths. Like OPL, it also has units of length*

For example: a wave passed through glass will appear to travel a greater distance than an identical wave in air.

$$\mathbf{OPD \equiv (n_2\, \boldsymbol{l}_2) - (n_1\, \boldsymbol{l}_1)}$$

Why???

This is because the source in the glass will have experienced a greater number of wavelengths due to the higher refractive index of the glass

Example:

Suppose a laser beam travels through a 10mm-thick glass window that has an index of refraction = 1.517. What is the OPL through this window?

$OPL = n_{glass}\ l_{glass}$

$= (1.517).\ 10\text{ mm} = 15.17\text{ mm}$

► In the time it takes the laser beam to travel 10mm through the glass window, it would have traveled 15.17mm through air.

Example:

what is the distance that this laser beam would travel through the air, in the same amount of time it takes it to travel through the actual window of a thickness 5 mm?)

OPL = $n_{glass}\ l_{\text{glass}}$

$= (1.517).\ 5\text{ mm} = 7.585\text{ mm}$

Example:

How long did it take this laser beam to travel through the 10mm-thick window?

t_{glass} = (distance / velocity)= $(l / v_{glass}) = (n_{glass}\ l_{\text{glass}} / c)$

$= (1.517x\ 0.1\ m)/ (3x10^8 m\ sec^{-1}) = 50.56$ *femtoseconds*

femtoseconds $= 10^{-11} sec$

Example:

How long did it take this laser beam to travel through the equivalent air-thickness (the OPL)?

$t_{\text{air}} = (L_{air} / c)$

$= (.01517\text{ m}) / (3x10^8 m\ sec^{-1}))$

$= 50.56$ *femtoseconds*

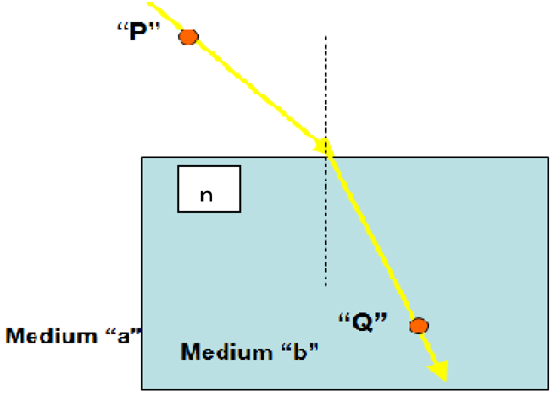
► *The same amount of time, by definition!*

Index of Refraction for Yellow Sodium Light $\lambda_0 = 589$ nm

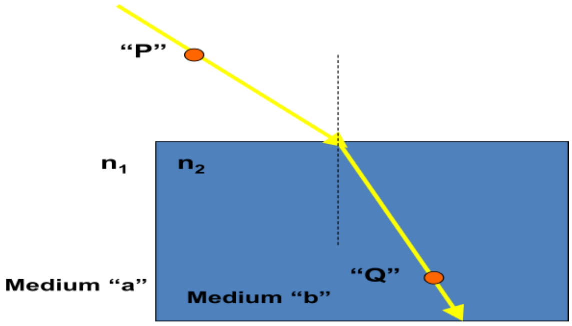
Substance	**Index of Refraction, n**
Solids	
Ice (H_2O)	1.309
Fluorite (CaF_2)	1.434
Polystyrene	1.49
Rock salt (NaCl)	1.544
Quartz (SiO_2)	1.544
Zircon $(ZrO_2 \cdot SiO_2)$	1.923
Diamond (C)	2.417
Fabulite $(SrTiO_3)$	2.409
Rutile (TiO_2)	2.62
Glasses (typical values)	
Crown	1.52
Light flint	1.58
Medium flint	1.62
Dense flint	1.66
Lanthanum flint	1.80
Liquids at 20°C	
Methanol (CH_3OH)	1.329
Water (H_2O)	1.333
Ethanol (C_2H_5OH)	1.36
Carbon tetrachloride (CCl_4)	1.460
Turpentine	1.472
Glycerine	1.473
Benzene	1.501
Carbon disulfide (CS_2)	1.628

- *Traveling through two different mediums with different velocities, the total time the ray travels from an arbitrary point **"P"** to another arbitrary point **"Q"** is:*

$$t = l_1 / v_1 + l_2 / v_2$$



Given, velocity is: $v = c / n$



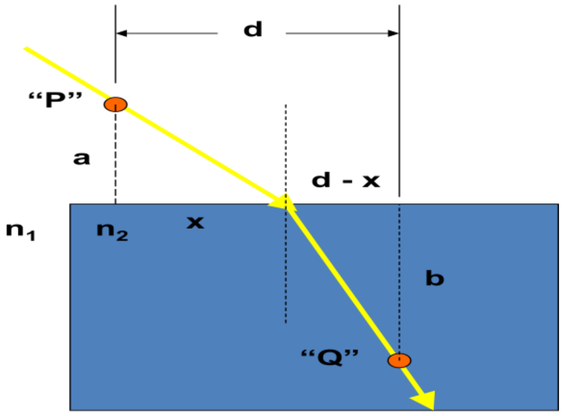
Then our equation becomes:

$$t = l_1 / (c/n_1) + l_2 / (c/n_2)$$

This can be rewritten as:

$$t = (n_1/c) * l_1 + (n_2/c) * l_2$$

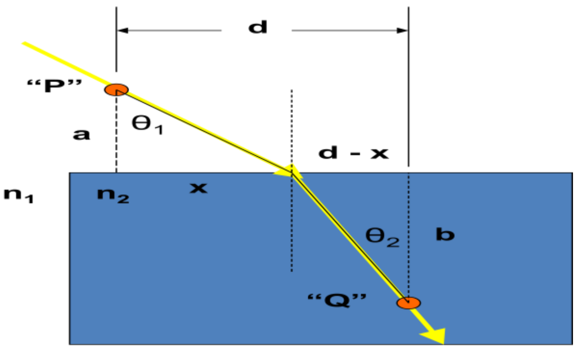
The distances r_1 and r_2 can be found by simple trigonometry.



The distance the ray travels is therefore the hypotenuse of two triangles.

$$l_1 = \sqrt{a^2 + x^2},\ l_2 = \sqrt{b^2 + (d - x)^2}$$

We assign "thetas" for the angles between the rays and the normal's to the surface.



Putting the two equations together, and differentiating it with respects to time yields:

$$\frac{dt}{dx} = \frac{n1}{c}\frac{d}{dx}\sqrt{a^2 + x^2} + \frac{n2}{c}\frac{d}{dx}\sqrt{b^2 + (d - x)^2}$$

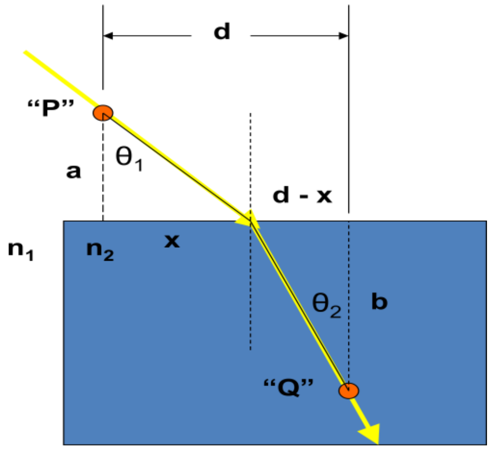
Deriving the equation gives:

$$\frac{dt}{dx} = \frac{n1}{c} * \frac{1}{2} * \frac{2x}{\sqrt{a^2+x^2}} + * \frac{n2}{c} * \frac{1}{2} * \frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}}$$

Simplifying and setting the equation equal to "0" yields:

$$\frac{n1x}{c\sqrt{a^2+x^2}} - \frac{n2(d-x)}{c\sqrt{b^2+(d-x)^2}} = 0$$

Recognizing the Trigonometric function of sines:



$\text{Sin}\theta_1$ = (opposite / hypotenuse) = $\frac{x}{\sqrt{a^2+x^2}}$

$\text{Sin}\theta_2$ = (opposite / hypotenuse) = $\frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$

Simplifying the equations yields (Snell's Equation for the Law of Refraction)

$$n_1 \text{Sin}\,\theta_2 - n_2 \text{Sin}\,\theta_2 = 0$$

Or

$$n_1 \text{Sin}\,\theta_2 = n_2 \text{Sin}\,\theta_2$$

N_University of Bagdad
College of Science
Department of Physics
Dr. Sarmed S. Alawadi

Subject: Optics I
Semester: First
Year: First year Physics

Chapter Two

Reflection and Refraction at Plane Surfaces

Reflection and Refraction at Plane Surface

When light hits a surface, one or more of three things may happen. The light may be - reflected back from the surface, transmitted through the material (in which case it will deviate(انحرف)from its initial direction, a process known as refraction) - or absorbed by the material if it is not transparent at the wavelength of the incident light.

The travel of light waves can often(غالبا) be approximate by straight lines (rays).The study of the properties of light with this approximation is called geometric optics.

The figure below shows an example of light waves traveling in approximately straight lines. A thin beam of light, angled upwards from the lower right and traveling through water, encounters a flat surface (plane) of intersection between water and air. Part of the light is reflected by the surface, forming a beam directed downward toward the left. This reflected beam travels as if it bounced from the surface. The other portion of light travels through the surface and into the air, forming a beam directed upward and to the left.

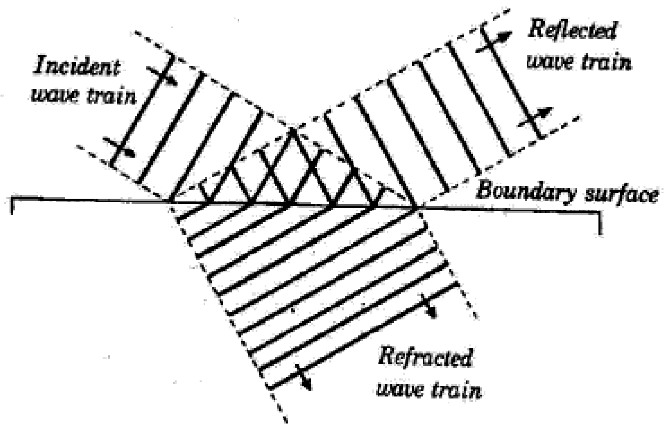


Fig. (2-1): Reflection and train of plane waves.

The travel of light through a surface (or interface) that separates two different materials is called refraction. The direction of the beam is “bent” at the surface as it travels from the water to the air, and is said to be refracted. Note that the bending occurs only at the surface.

Reflection of Light

1. When light travels from one medium to another, part of the light can be reflected at the media interface.i.e: at the boundary between two media, the light ray can change direction by reflection, this means it is directed backward into the first medium or refraction

   a) Reflection off of a smooth surface and mirrors is called **specular(براق) reflection** *"which can be described as reflection without scattering"*. Some examples of specular reflectors are the surfaces of many types of glass, polished metals and the undisturbed surfaces of liquids. Some of these, such as glass and many liquids, also transmit light, whereas light does not penetrate beyond the surface of a metal. The fact that light is not transmitted through metals can be explained in terms of the interaction between the light and electrons within the metal.

The incident light on a smooth can be represented by a bundle(bundle) of parallel rays. The reflected light will also travel in a well-defined direction which can be represented using another bundle of parallel rays. Since there is no scattering, for each incident ray there is only one reflected ray.

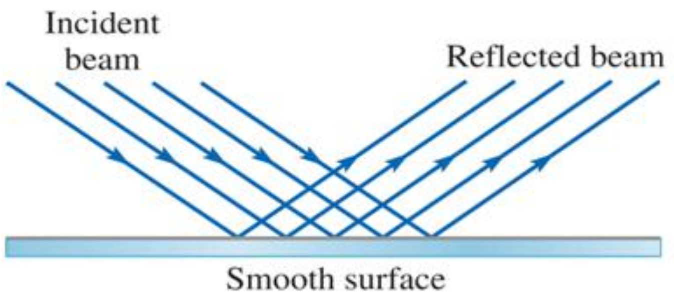


Fig. (2-2)

a) Reflection off of a rough surface (not shiny) is called diffuse reflection. In diffuse reflection, a single ray of light scatters into many directions.

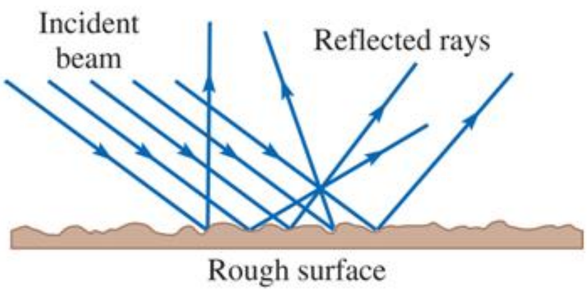


Fig. (2-3)

Law of Reflection

In order to describe the relation between reflected and incident rays we need to look at the point where the incident ray meets the reflecting surface. At that point we imagine a line constructed perpendicular to the surface, called the normal to the surface. The reflected ray also departs from the same point. The angle between the incident ray and the normal is called the angle of incidence θ_1 and the angle between the normal and the reflected ray is called the angle of reflection θ_1'. The behavior of the rays in specular reflection can be described completely by two laws, illustrated in figure (2-4):

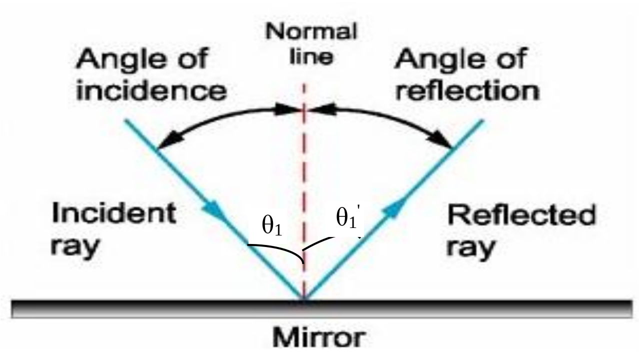


Fig. (2-4)

1. The incident ray, the normal and the reflected ray all lie in one plane.
2. The angle of incidence is equal to the angle of reflection.

$$\theta_1 = \theta_1'$$

Law of reflection: *a plane wave is reflected from a plane surface with the angle of reflection equal to the angle of incidence.*

❖ Note that the amount of light reflected cannot be predicted(توقع) from these laws. That depends on the reflectivity of the surface.

Refraction of Light

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, part of the ray is reflected and part of the ray enters the second medium. The ray that enters the second medium is bent at the boundary. This bending of the ray is called refraction. The angle of refraction, θ_2, depends on the properties of the medium.

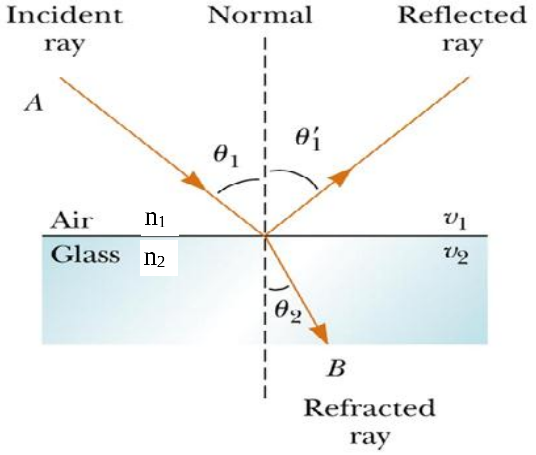


Fig. (2-5)

Law of refraction: *at the boundary between any two given materials; the ratio of the sine of the angle of incidence to the sine of the angle of refraction is equal to the ratio of the velocities in the two media.*

Since by definition:

$$n_1 = c / v_1 \quad \text{and} \quad n_2 = c / v_2 \qquad (2\text{-}1)$$

It follows that:

$$v_1 / v_2 = n_2 / n_2 \qquad (2\text{-}2)$$

Equation (2-2) may be written as:

$$\frac{\sin\varphi_1}{\sin\varphi_2} = \frac{n_2}{n_1} \qquad (2\text{-}3)$$

Or

$$n_1 \sin\phi_1 = n_2 \sin\phi_2 \qquad (2\text{-}4)$$

This is called as **Snell's law**. It follows from the low of refraction that:

$$\frac{\sin\varphi_1}{\sin\varphi_2} = a \quad \text{constant} \qquad (2\text{-}5)$$

The constant is known as *the refractive index of material (2) with respect to material (t).*

$$\frac{\sin \varphi_1}{\sin \varphi_2} = {}_1n_2 \qquad \textbf{(2-6)}$$

It can be shown that:

$${}_1n_2 = \frac{\textit{velocity of light in material (1)}}{\textit{velocity of light in material (2)}} \qquad \textbf{(2-7)}$$

When light is travelling from material (2) to material (1) we use the refractive index of (1) with respect to (2), ${}_2n_1$. It follows from above equation that:

$${}_1n_2 = \frac{1}{{}_1n_2} \qquad \textbf{(2-8)}$$

It is evident from Snell's law that:

1. Rays of light passing from a medium of smaller refractive index into one of larger index **($n_1 < n_2$)**, as from air into glass, Fig. (2-6):

- ❖ Light may refract into a material where its speed is lower
- ❖ The angle of refraction is always less than the angle of incidence, **($\phi_1 > \phi_2$)**
- ❖ The ray bends ***toward*** the normal

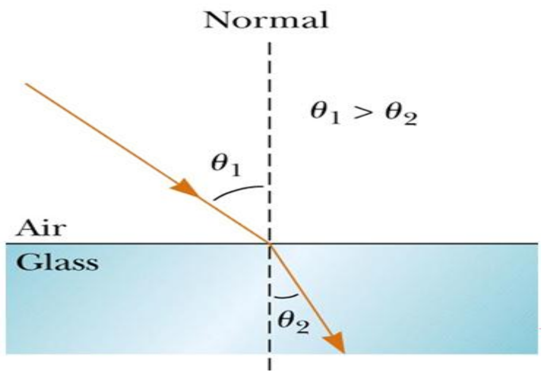


Fig. (2-6)

2. Rays of light passing from a medium of large refractive index into one of smaller index **($n_2 > n_1$)**, as from glass into air, i.e. travelling in the opposite direction, Fig. (2-7):

- ❖ Light may refract into a material where its speed is higher
- ❖ The angle of refraction is greater than the angle of incidence, **($\phi_1 < \phi_2$)**
- ❖ The ray bends ***away from*** the normal

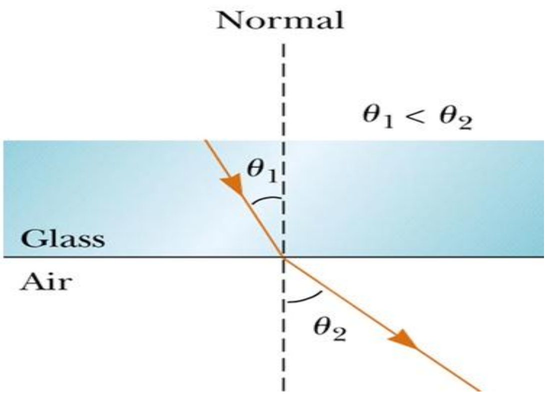


Fig. (2-7)

Examples

1. Find the angle of refraction: (a) when a ray of light is travelling from air to glass at an angle of incidence of 40^o, (b) when a ray of light is travelling from glass to air at an angle of incidence of 20^o. (Refractive index of glass with respect to air = 1.5).

Solution:

(a) By Snell's law:

$$\frac{\sin\varphi_1}{\sin\varphi_2} = {}_1n_2$$

The angle of incidence $\phi_1$is 40^o and therefore:

$$\frac{\sin 40^o}{\sin\phi_2} = 1.5$$

$$\sin\phi_2 = \frac{\sin 40^o}{1.5} = 0.4285$$

$$\phi_2 = 25.4^o$$

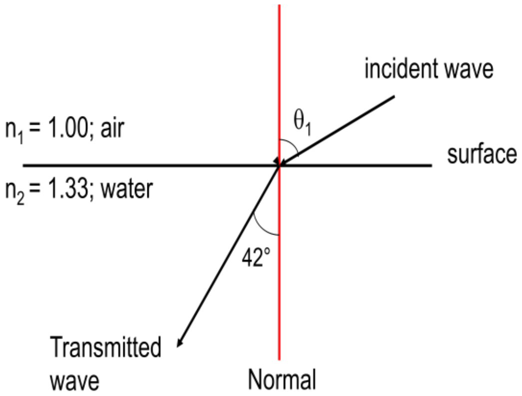
(b) The angle of incidence ϕ_1 is 20^o. In this case light is travelling <u>from glass to air</u> and therefore we require the refractive index of <u>air with respect to glass</u>. This is the reciprocal of the refractive index of glass with respect to air and therefore:

$$\frac{\sin 20^o}{\sin\phi_2} = \frac{1}{1.5}$$

$$\sin\phi_2 = 1.5 \sin 20^\circ = 0.513$$

$$\phi_2 = 30.9^o$$

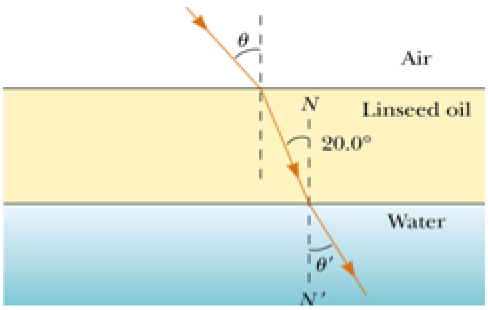
2. Sunlight strikes the surface of a lake. A diver sees the Sun at an angle of 42.0° with respect to the vertical. What angle do the Sun's rays in air make with the vertical?



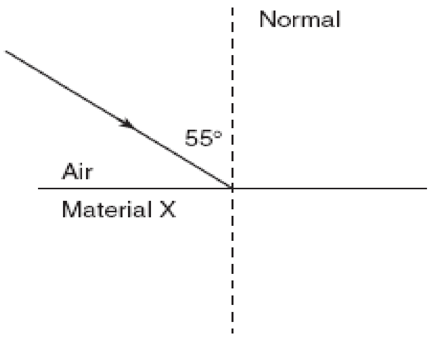
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$(1.00)\sin\theta_1 = (1.333)\sin 42°$$
$$\sin\theta_1 = 0.8920$$
$$\theta_1 = 63.1°$$

Problems

1. What is the speed of light in quartz? Answer: 1.97 x 10^8 m/s.
2. What is the angle of refraction when a ray from air with an angle of incidence of 25 ° is incident to water? Draw the ray diagram. Answer: 18.5 °.
3. The light shown in the figure makes an angle of 20° with the normal NN' in the linseed oil (n=1.48). Determine angles θ and θ'.



4. A monochromatic light ray f= 5.09 x 10^{14} Hz is incident on medium X at 55°. The absolute index of refraction for material X is 1.66. What is material X? Determine the angle of refraction. Determine the speed of light in medium X.



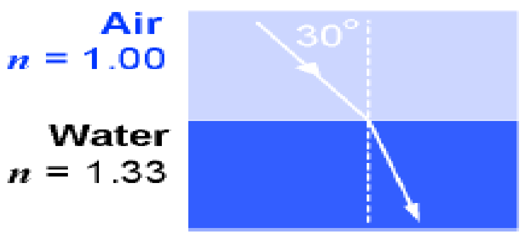
The index of 1.66 is Flint Glass. To find the angle of refraction use Snell's Law.

$$\theta_2 = 30^\circ$$

To find the speed use n=c/v.

$$v = 1.8 \times 10^8 \text{ m/s}$$

5. A ray of light traveling through air is incident on a smooth surface of water at an angle of 30° to the normal. Calculate the angle of refraction for the ray as it enters the water.



Total Internal Reflection (T.I.R.) and Critical Angle

If the light is propagating from the denser medium to the less dense medium $n_1 > n_2$, at a large angle of incidence. For example: light going from glass-to- air or from water- to- air. It is possible for the angle of incidence to be such that the angle of refraction is 90°, fig. (2-8b).

The angle of incidence at which this happens is called the ***critical angle (c)***, fig. (2-8b). From Snell's law;

$$n_1 \sin 90 = n_2 \sin c \qquad \textbf{(2-9)}$$

Critical angle: *It is the angle of incidence in the optically dense medium which have right refraction angle (90°).*

Since sin 90°=1

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin\theta_c = n_2 \sin 90^\circ = n_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}(n_2 / n_1) \qquad \textbf{(2-10)}$$

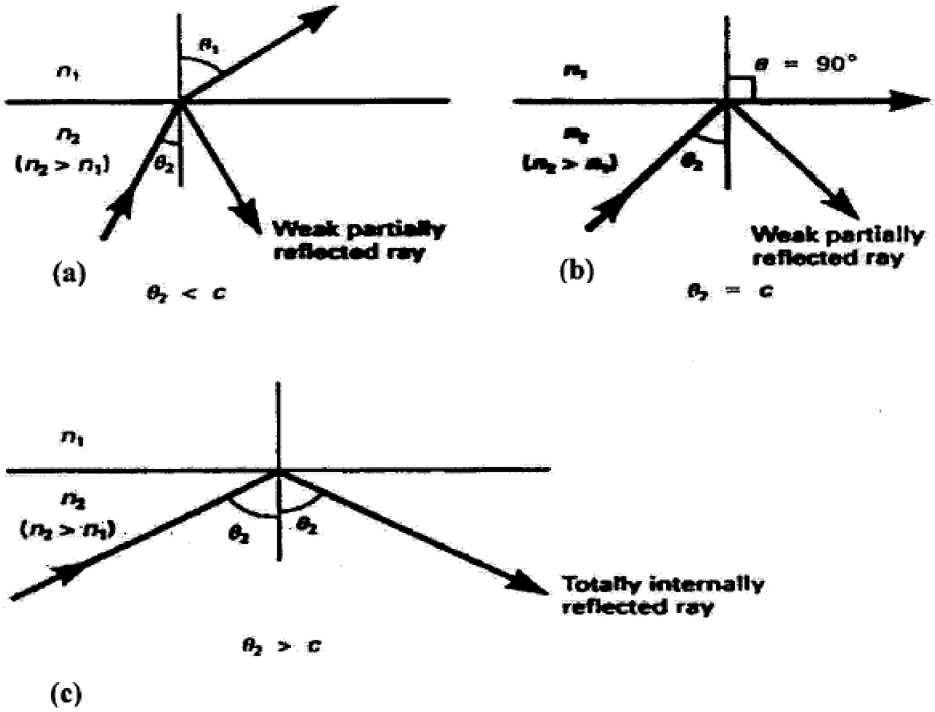


Fig. (2-8): Three possibilities when light travels towards a less dense medium.

- If ϕ < **c**, the light is refracted in the normal way, fig. (2-8a).
- If ϕ_1 > **c**, the light is totally reflected back into material 2, fig. (2-8c).

  This is known as ***Total Internal Reflection (T. I. R)***.
- Total internal reflection cannot occur when the light is travelling towards an optically more dense material.

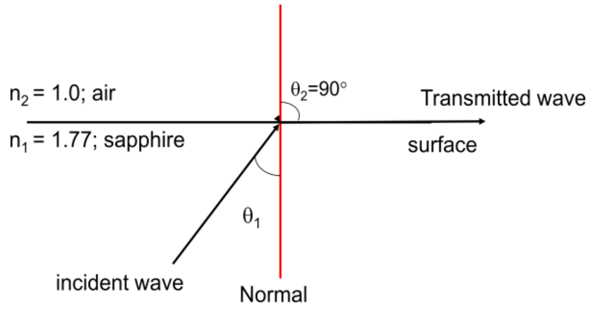
- Total internal reflection is different from ordinary reflection in that it really is 'total'; there are no losses and the totally internally reflected light has the same intensity as the incident light.

- Fiber optic cables rely on the principle of total internal reflection at the glass/air interface to transmit light over long distances with almost zero losses.

Problems

1- Determine the critical angle for both water and diamond with respect to air.

2- Calculate the critical angle for sapphire surrounded by air.

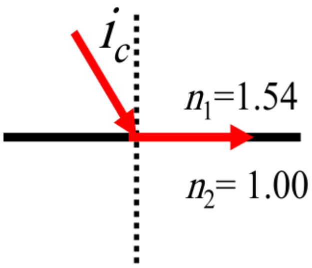


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$(1.77)\sin\theta_c = (1.00)\sin 90°$$
$$\sin\theta_c = 0.565$$
$$\theta_c = 34.4°$$

3- The refractive index of diamond with respect to air is 2.42. Calculate the critical angle for diamond-air boundary.

$$\theta_c = \sin^{-1}(n_2/n_1)$$
$$= \sin^{-1}(1/2.42)$$
$$= \sin^{-1}(0.413)$$

4- Find the critical angle between a water-diamond interfaces.



$$n_1 \sin i = n_2 \sin r$$
$$1.54 \sin i_c = 1.00 \sin 90$$
$$\sin i_c = 1.00/1.54$$
$$i_c = \sin^{-1}.649 = 40°$$

5- Light travels from air into an optical fiber with an index of refraction of 1.44:

(a) In which direction does the light bend?

(b) If the angle of incidence on the end of the fiber is 22°. What is the angle of refraction inside the fiber?

(c) Sketch the path of light as it changes media

(a) Since the light is traveling from a rarer region (lower n) to a denser region (higher n), it will bend toward the normal.

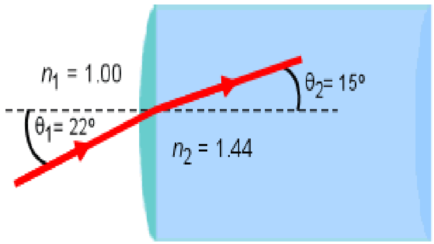
(b) $n_1 \sin i = n_2 \sin r$

$(1.00) \sin 22^o = 1.44 \sin r.$

$\sin r = (1.00/1.44) \sin 22^o = 0.260$

$r = \sin\text{-}1\ (0.260) = 15^o.$

(c)



Refraction by plane parallel plate

When a ray of light is refracted successively at two parallel plane surfaces, it passes from the first medium into the second and passes into the first again. This is shown in fig. (2-9), where the ray of light actually coming from object O appears to come from image I along a line parallel to its original direction of travel.

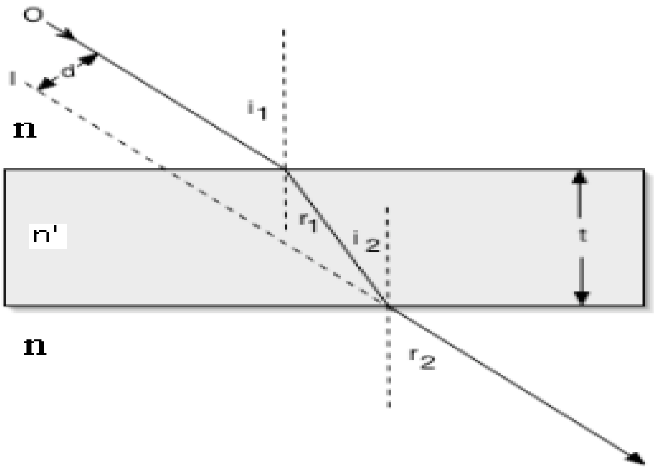


Fig. (2-9)

Let i_1 and r_1 be the angle of incidence and refraction at the upper surface, and i_2 and r_2 be the angle of incidence and refraction at the lower surface.

Let n be the index of the medium on either side of plate, and the index of the plate be n', n'> n; and from Snell law:

$$n \sin i_1 = n' \sin r_1$$

$$n' \sin i_2 = n \sin r_2$$

But as is evident from the diagram

$$r_1 = i_2$$

Combining these relations, we find:

$$i_1 = r_2$$

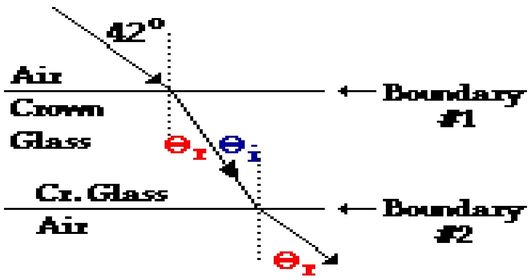
The important concept is this: When light approaches a layer which has the shape of a parallelogram that is bounded on both sides by the same material, then the angle at which the light enters the material is equal to the angle at which light exits the layer. i.e: that is:

1- The emergent ray is parallel to the incident ray.
2- It is not deviated in passing through the plate but is displace by the distance d, which equal to:

$$d = t\,\frac{\sin(i_1 - r_1)}{\cos(r_1)}$$

Problem

A ray of light in air is approaching the boundary with a layer of crown glass at an angle of 42 degrees. Determine the angle of refraction of the light ray upon entering the crown glass and upon leaving the crown glass.



Boundary 1

1.00 sin (42) = 1.52 sin(r)

0.669 = 1.52 sin (r)

0.4402 = sin (r)

$\sin^{-1}$ (0.4402) = 26.1

r =26.1

Boundary 2:

1.52 sin (26.1) = 1.00 sin(*r*)

1.52 (0.4402) = 1.00 sin (*r*)

0.6691 = sin (r)

sin-^{1} (0.6691) = r

r = 42.0 degrees

Refraction by a prism

The prism is one or another of its many forms, is second only to the lens as the most useful single piece of optical apparatus.

Consider a light ray incident at an angle i on one face of a prism as in fig. (2-10). Let the index of the prism be n. The included angle at the apex be A, and let the medium on either side of the prism be air. It is desired to find the angle of deviation, D.

Deviation by a Prism

The deviation produced by a prism depends on the angle at which the light is incident on the prism. It can be shown, that the deviation is a minimum when the light passes symmetrically through the prism. The situation is shown in fig. (2-10), where D_m is the minimum deviation.

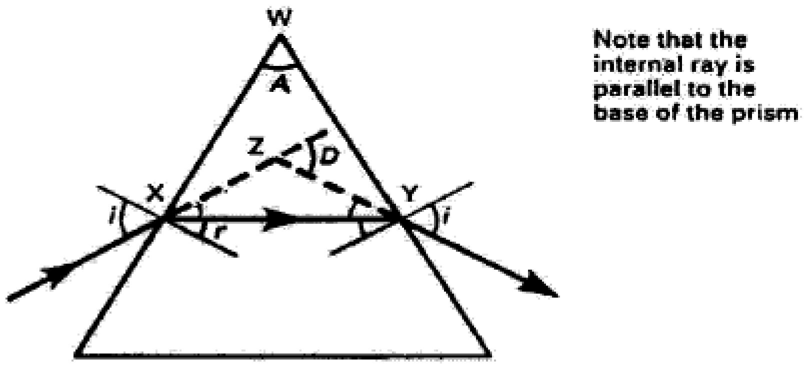


Fig. (2-10): Minimum deviation by a prism.

In Δ XWZ:

$D_m = ZXY + ZYX$

$D_m = (i - r) + (i - r)$

$D_m = 2i - 2r \qquad$ **(2-11)**

In Δ WXY:

$180^o = A + WXY + WX$

$180^o = A + (90^o - r) + (90^o - r)$

$A = 2r \qquad$ **(2-12)**

Adding eqs. (2-11) and (2-12) leads to:

$$i = \frac{A + D}{2}$$

From eq. (2-12):

$$r = \frac{A}{2}$$

From Snell's law:

$$n_1 \sin i = n_2 \sin r$$

$$n_1 \sin \frac{A + D_m}{2} = n_2 \sin \frac{A}{2}$$

If the light is incident from air, $n_1=1$, and therefore:

$$n_2 = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}$$

If the angle of prism is small, the angle of minimum deviation is small also and we may replace by the sins of the angles by the angles.

$$n_2 = \frac{\frac{A + D_m}{2}}{\frac{A}{2}}$$

$$D_m = (n - 1) A$$

where n_2 is the refractive index of the material of the prism.

Dispersion by a Prism

When a narrow beam of white light is refracted by a prism, the light spreads into a band of colors - the ***spectrum*** of the light. The effect was first explained by **Newton**. He identified the colors as ranging from red at one side of the band, through orange, yellow, green, blue, and indigo, to violet at the other.

The index of refraction in anything except a vacuum depends on the wavelength of the light. This dependence of n on λ is called dispersion. The variation in refractive index with color is called **dispersion** because the speed of light is slightly different in glass for each frequency of light ($n=c/v$). (In vacuum all colors have speed $c=3x10^8$m/s).

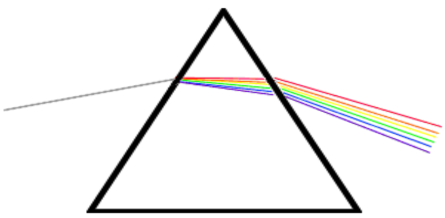


Fig. (2-11): In a prism, material dispersion causes different colors to refract at different angles, splitting white light into a rainbow.

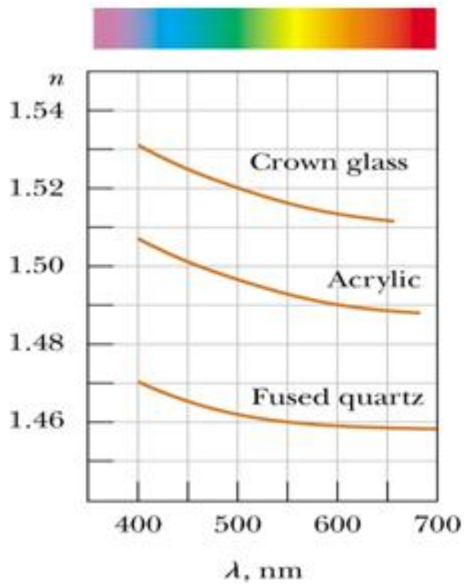


Fig. (2-12)

***Dispersion** is the separation of white light into its component colors*. It is plays a very important role in fiber optical communications.

- The index of refraction for a material usually decreases with increasing wavelength ('normal' dispersion)
- Violet light refracts more and red least light when passing from air into a material (through a glass prism). Colors between blue and red are bent proportional to their position in the spectrum.

The light is said to be dispersed into a spectrum and ***"the difference between the angles of deviation of any two rays is called the angular dispersion of those particular rays".***

The ***dispersive power ω*** is ***"the ability of the optical elements (prism) to disperse the white light to it's components"***, and is given by the relation:

$$\omega = \frac{n_B - n_R}{n_Y - 1} \qquad (2\text{-}15)$$

Where n_B, n_R, and n_Y are the indices for blue, red, and yellow position of spectrum respectively.

Example

The dispersive power of a silicate flint glass for which $n_B = 1.632$, $n_Y = 1.62$, and $n_R = 1.613$ is:

$$\omega_{Flint} = \frac{1.632 - 1.613}{1.62 - 1} = 0.031$$

While that of a silicate crown glass for which n_B=1.513, n_Y=1.508, and n_R = 1.504 is:

$$\omega_{Crown} = \frac{1.513 - 1.504}{1.508 - 1} = 0.018$$

The angle δ considered the **mean deviation of the spectrum**, while *"the difference between tow mean deviation angles, such as* $\delta_B - \delta_R$ *is a measure of the* ***dispersion*** *of the spectrum".*

The dispersion is small if the angle of the prism is small, then for example:

$$\delta_B = (n_B - 1)\, A$$
$$\delta_R = (n_R - 1)\, A \qquad (2\text{-}16)$$
$$\delta_Y = (n_Y - 1)\, A$$

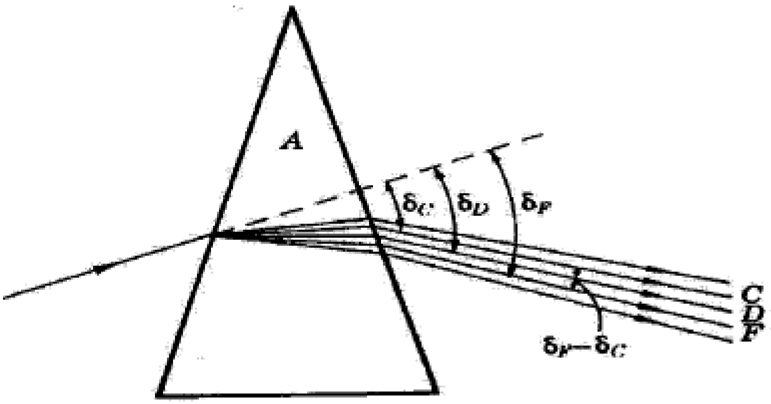


Fig. (2-13): Angles of deviation for the rays.

The dispersion, $\delta_B - \delta_R$, is therefore:

$$\delta_B - \delta_R = (n_B - 1)\, A - (n_R - 1)\, A = (n_B - n_R)\, A$$

And the ratio of the dispersion to the mean deviation is:

$$\frac{\delta_B - \delta_R}{\delta_Y} = \frac{(n_B - n_R)A}{(n_Y - 1)A} = \frac{n_B - n_R}{n_Y - 1} = \omega \qquad (2\text{-}17)$$

*The **dispersive power** is equal to the ratio of the dispersion to the mean deviation, when light is dispersed at minimum deviation by a prism of small angle.*

Example

Compute the mean deviations and dispersions produced by the flint and crown glasses above, if the prism angle is l0°.

Solution:

For the flint glass:

Mean deviation = δ_Y = $(n_Y - 1)$ A = (1.62 - l) 10 = 6.2°

Dispersion = $\omega\,\delta_Y$ = 0.03l x 6.2 = 0.192.

For the crown glass:

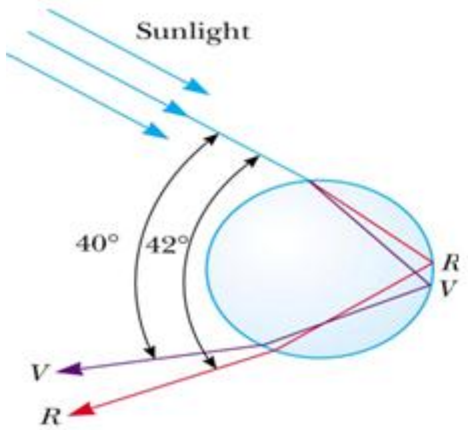
Mean deviation = δ_Y = $(n_Y - 1)$ A = (1.508 - l) 10 = 5.08°

Dispersion = $\omega\,\delta_Y$ = 0.0l8 x 5.08 = 0.0191.

The Rainbow

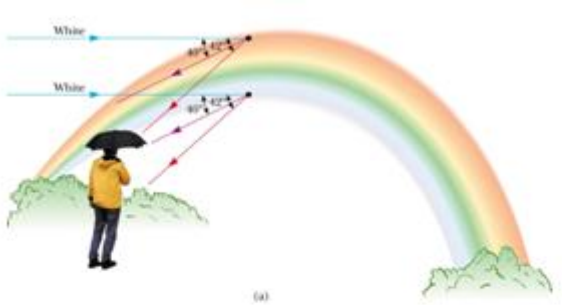
The separation of colors by a prism is an example of dispersion. At the surfaces of the prism, Snell's law predicts that light incident at an angle θ to the normal will be refracted at an angle arcsin (sin (θ) / n). Thus, blue light, with its higher refractive index, is bent more strongly than red light, resulting in the well known **rainbow** pattern.

- The rainbow takes the concept of dispersion and multiples it through the atmosphere
- The sun shines on water droplets in a cloud or when it is raining
- The light is dispersed by the raindrop into its spectral colors



- At the back surface the light is reflected
- It is refracted again as it returns to the front surface and moves into the air
- The rays leave the drop at various angles
- The angle between the white light and the violet ray is 40°
- The angle between the white light and the red ray is 42°

Observing the Rainbow



(a)

- If a raindrop high in the sky is observed, the red ray is seen
- A drop lower in the sky would direct violet light to the observer
- The other colors of the spectra lie in between the red and the violet

Problem

The index of refraction for crown glass for red light is 1.514.What is the speed of red light in crown glass?   Answer: 1.98×10^{8} m/s

University of Bagdad
College of Science
Department of Physics
Dr.Sarmed S. Alawadi

Subject: Optics I
Semester: first
Year: First year

Chapter Three

Reflection and Refraction at Spherical Surfaces

3-1 Introduction

Most of lenses and mirrors are spherical or plane, because only spherical and plane surfaces can be produced by machine methods at reasonable cost. When a train of light waves passes through an optical instrument, the curvature of the wave fronts is altered at each boundary surface.

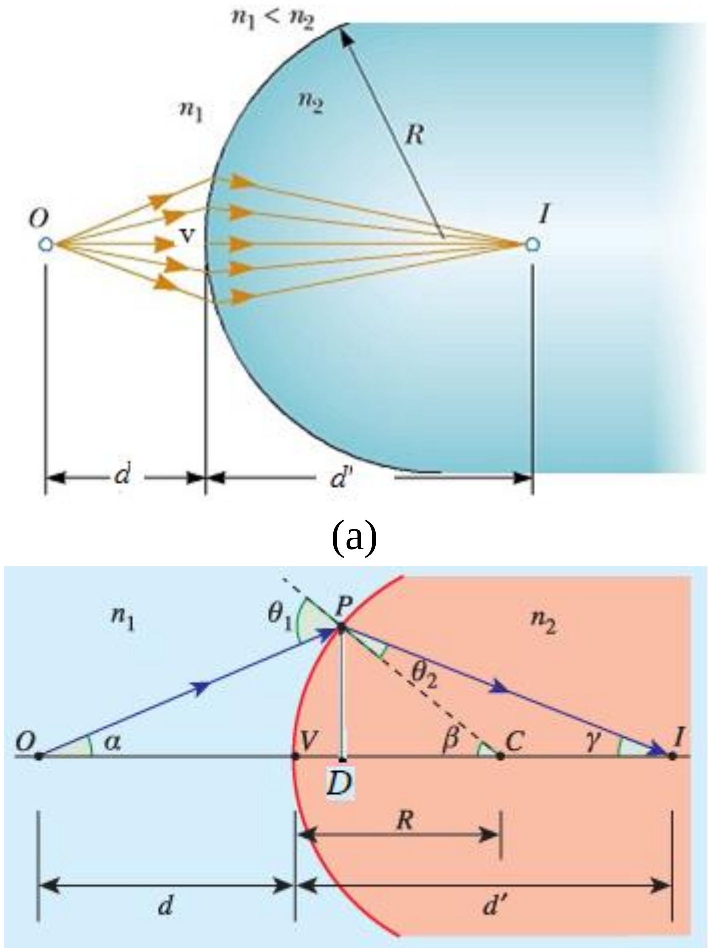
A ray, in its passage (فقرة) through an optical instrument, is made up of a number of segments (القطع)of straight lines, deviated at reflecting or refracting surfaces by angles which can be computed from the law of reflection or from Snell's law. The problem of tracing (البحث عن) the path of a ray reduces to a problem in geometry and this branch of optics is called geometrical optics.

3-2 Reflection at Spherical Surfaces

Figure (3-1) shows a spherical surface with radius, *R* forms an interface between two media with refractive indices *n1* and *n2*.

- A line *OVC* through the center of curvature is called axis.
- The point *V*, where the axis intersects (يتقاطع) the surface, is called the vertex (قمة الرأس).
- Fig. (3-1a) shows a pencil of rays diverging from apoint *O*, called the *object*.

- The surface forms an image *I* of a point object *O* as shown in figure (3-1b).
- The distance from the object to the vertex is called the *object distance* and is represented by *d*. The distance *d'* is the *image distance*.



(a)

(b)

Fig. (3-1): Refraction of rays at a spherical surface.

- The incident ray *OP* makes an angle α with the axis and is incident on the refracting surface at point, *p*.
- Point *C* is the centre of curvature of the spherical surface and *PC* is normal.
- The incident ray *OP* making an angle $\theta_1$with the normal and is refracted to ray *PI* making an angle θ_2 where $n_1<n_2$.

- The refracted ray crosses the axis at point *I* at a distance *d'* to the right of the vertex, and makes an angle γ with the axis.

- For small values of the angle α all rays from *O* intersect at *I*, which is called the image of the point *O*.

From the figure, the triangle *OPC* and the law of sines:

اي بتطبيق قانون الجيوب على المثلث OPC

$$\frac{\sin(\pi - \theta_1)}{R + d} = \frac{\sin \alpha}{R}$$

or

$$\frac{\sin(\pi - \theta_1)}{\sin \alpha} = \frac{R + d}{R}$$

حيث ان الزاويا المكملة لزاوية معينة يساوي جيب الزاوية نفسها اي ان:

And since $\sin(\pi - \theta_1) = \sin\theta_1$,

So

$$\sin\theta_1 = \frac{R+d}{R}\sin\alpha \qquad (3\text{-}1)$$

The angle of refraction θ_2 may found by Snell's law.

$$\sin\theta_2 = \frac{n1}{n2}\sin\theta_1 \qquad (3\text{-}2)$$

Since the slop of the ray *PI* is negative with respect to the axis, then the angle γ is negative. And the sum of the inside angles of the triangle *OPI* equal to π

$$\alpha + (\pi - \theta_1) + \theta_2 + (-\gamma) = \pi$$

$$\alpha + (\pi - \pi) - \theta_1 + \theta_2 = \gamma$$

$$\theta_2 + \alpha - \theta_1 = \gamma \qquad (3\text{-}3)$$

The image distance d' is found from the triangle PCI, by the law of sines to be:

$$\frac{-sin\gamma}{R} = \frac{sin\theta_2}{d' - R}$$

So the images distance:

$$d' = R - R\frac{sin\theta_2}{sin\gamma} \qquad (3\text{-}4)$$

From the figure,

$$\Delta POC \rightarrow \theta_1 = \alpha + \beta \qquad (3\text{-}5)$$

$$\Delta PIC \rightarrow \beta = \gamma + \theta_2$$

$$\theta_2 = \beta - \gamma \qquad (3\text{-}6)$$

By using ΔPOD, ΔPCD and ΔPID thus

$$\tan\alpha = \frac{PD}{DO};\ \tan\beta = \frac{PD}{DC};\ \tan\gamma = \frac{PD}{ID}$$

By considering point P very close to the pole V or; all angles that rays make with surfaces are small, take approximation $\sin\theta \approx \theta$ known as the *paraxial region; so:*

$$.sin\ \theta_1 = \theta_1;\ sin\ \theta_2 = \theta_2$$

and

$$;tan\alpha = \alpha;\ tan\beta = \beta; tan\gamma = \gamma$$

$$OD \approx OV = d\ ;\ CD \approx CV = R\ ;\ ID \approx IV = d'$$

And Snell's law can be written as

$$n_1\ \theta_1 = n_2\ \theta_2 \qquad (3\text{-}7)$$

By substituting eq. (3-5) and (3-6) into eq. (3-7), thus

$$n_1(\alpha + \beta) = n_2\ (\beta - \gamma)$$

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

Then

$$n_1\frac{PD}{d} + n_2\frac{PD}{d'} = (n_2 - n_2)\frac{PD}{R}$$

$$\boxed{\frac{n_1}{d} + \frac{n_2}{d'} = \frac{(n_2 - n_1)}{R} \Rightarrow \quad (3\text{-}8) \quad \textit{Equation of spherical refracting surface}}$$

Where

d' : image distance from pole

d: object distance from pole

n_1: refractive index of medium 1 (Medium containing the incident ray)

n_2: refractive index of medium 2 (Medium containing the refracted ray)

Note :

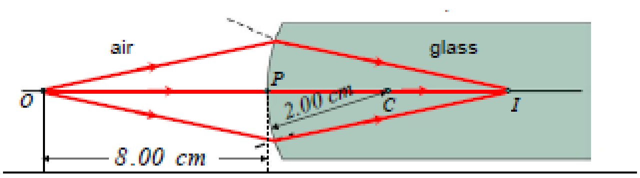
- **If the refraction surface is flat (plane)**

R=∞

$$\frac{n_1}{d} + \frac{n_2}{d'} = 0$$

Examples:

1-A cylindrical glass rod in air has refractive index of 1.52. One end is ground to a hemispherical surface with radius, R =2.00 cm as shown in figure below.



Find,

a. the position of the image for a small object on the axis of the rod,8.00 cm to the left of the pole as shown in figure.
b. the linear magnification.(Given the refractive index of air , *na*= 1.00)هذا الفرع سابق لاوانه

2- The end of a solid glass rod of index 1.50 is ground and polished to a hemispherical surface of radius 1 cm. A small object is placed in air on the axis 4 cm to the left of the vertex. Find the position of the image. Assume $n = 1.00$ for air.

3- A concave surface with a radius of 4 cm separates two media of refractive index $n = 1.00$ and $n' = 1.50$. An object is located in the first mediumat a distance of 10 cm from the vertex. Find *(a)* the primary focal length, *(b)* the secondary focal length, and *(c)* the image distance.

3-3 CONVENTION (اتفاقية)OF SIGNS

It is necessary a convention of signs for distances and angles. Many such conventions are in use and all have some points in their favor (صالح). Most of us use the following set of conventions:

1. All figures are drawn with the light traveling from *left to right*.
2. Consider object distances *(d) positive* when the object lies at the *left* of vertex of the refracting or refracting surface.
3. Consider image distances *(d') positive* when the image lies at the *right* of vertex of the refracting or refracting surface.
4. Consider radii of curvature *(R) positive* when the center of curvature lies at the *right* of the vertex Vice versa. [or : All convex surfaces are taken as having a positive radius, and all concave surfaces are taken as having a negative radius].
5. Consider angles *positive* when the slope of the ray with respect to the axis (or with respect to a radius of curvature) is *positive*.
6. Consider transverse dimensions *positive* when measured *upward* from the axis.
7. A *positive* value of the image distance d' indicates a *real* image, while a *negative* value indicates a *virtual* image.

3-4 Reflection at Spherical Surface

When light is incident on a surface bounding two transparent media of different indices of refraction, some light is reflected at the surface, with the angle of reflection of any ray equal to the angle of incidence. A ray such as *OP* in fig. (3-1a) gives rise to a reflected ray as well as a refracted ray. If the surface is convex (محدب) toward the left, as in fig. (3-2), the reflected rays appear to diverge from a virtual image of *O* at Q_1.

The angle between the reflected ray and the normal is found from the law of reflection instead of from Snell's law as in eq. (3-2).

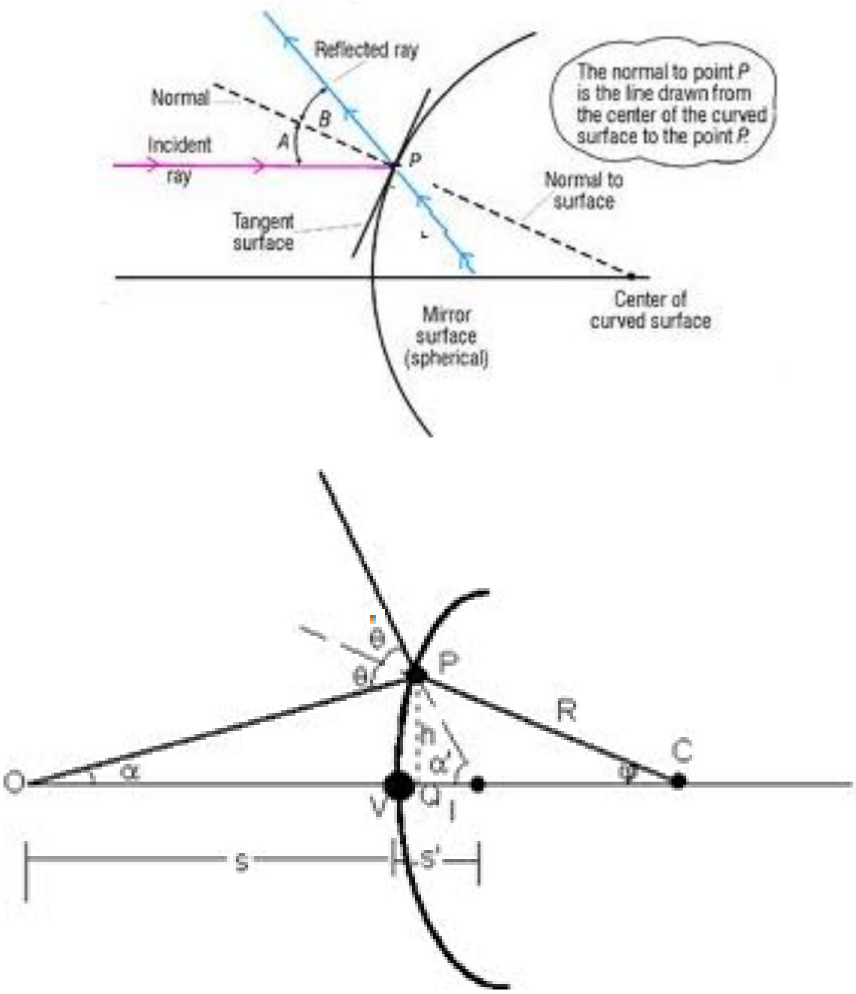


Fig. (3-2): Reflection at a spherical surface.

The angle of incidence θ of a ray from O in fig. (3-2) given by eq. (3-1). That is, if θ is the angle of reflection instead of eq. (3-2),

$$\theta = -\theta'$$

The negative sign enters since the reflected ray has a negative slope with respect to the normal. The same formulas that were derived for refraction can be used for reflection also. If we assume in Snell's law that $n_1 = n_2$, this law reduces to:

$$\sin\varphi_1 = -\sin\varphi_2$$
$$\varphi_1 = -\varphi_2$$

If we let $n_1 = n_2$ and let the primed quantities in eqs. (3-1) to (3-8) apply to the reflected as well as the refracted rays, these equations can be used for reflecting as well as for refracting surfaces.

A *virtual(الظاهري) image* formed by a mirror lies at the *right* of the reflecting surface, so that a positive value of *d'* corresponds to a *virtual image* and a *negative* value of *d'* to a *real image*.

As an illustration, consider the formation of an image by plane mirror.

- ❖ If the radius of curvature of such a mirror is infinite and setting $R = \infty$ and $n_1 = -n_2$ in eq. (3-8), we find:

$$d = d'$$

Since the object distance *d* is positive if the object is real, the image distance *d'* is positive also. Therefore

- a plane mirror forms a virtual image of a real object, at the same distance behind the mirror that the object is in front it.

❖ If the radius of curvature of the mirror is finite, eq. (3-8) becomes:

$$\frac{1}{d} + \frac{1}{d\prime} = -\frac{2}{R} \qquad (3\text{-}9)$$

3-5 Mirrors

A mirror is an object that reflects light and it's simply a piece of glass polished metal surface. In the past they were usually made by coating glass with silver (because of it is high efficiency in the UV and IR).

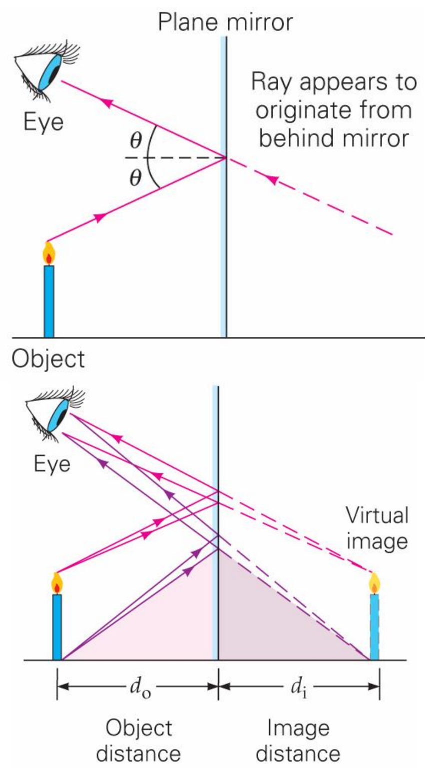
Now a day, vacuum evaporated coatings of aluminum on highly polished substrates have become the accepted standard for quality mirrors.

3-5-1 Plane Mirrors

Plane mirrors are ground to be flat – the flatter the more expensive. (Typically good ones have - where we use visible radiation- no hills or valleys larger than 500nm).

❖ 1. Images formed by plane (i.e., flat) mirrors have the following properties:

a) The image formed by a plane mirror is upright, identical in size to the object, and as far behind the mirror as the object is in front of it.



b) The image is unmagnified, virtual, and erect.

❖ 2. Image orientation(اتجاه):

a) Erect(اركت:معتدل): Image is oriented the same as the object.

b) Inverted(مقلوب): Image is flipped 180^0 with respect to the object.

❖ 3. Image classification:

a) Real: Image is on the same side of mirror as the object ⇒light rays actually pass through the image point.

b) Virtual: Image is on the opposite side of mirror from object ⇒ light rays appear to diverge from image point [i.e Light rays reflect on a plane mirror, and produce a virtual image behind the mirror. What's a virtual image? It means the light rays are NOT coming from a real point, there is no light where the image appears. Or in other word **the image is called virtual because it does not really exist behind the mirror**]

the properties of plane mirror image is:

The image distance equals the object distance.
• The image is unmagnified.
• The image is virtual.
• The image is not inverted (مقلوبة).
• Left and right are reversed (عكوسة)
**The intensity of the reflected beam depends upon the angle of incidence and the indices of refraction and they type of coating.

❖ 4. Image size is determined by the magnification of an object which is given by

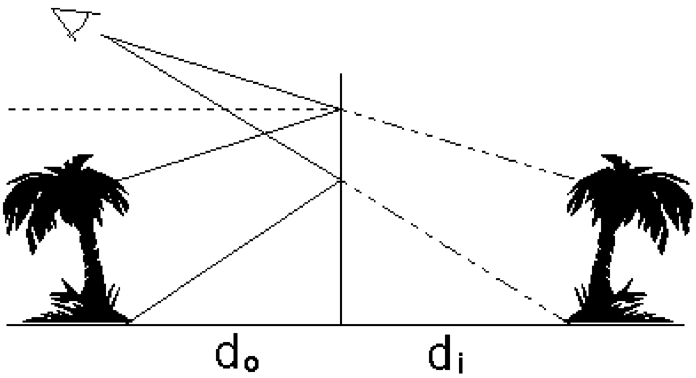
$$M = \frac{image\ height}{object\ height} = \frac{h'}{h}$$

h'= image height

h= object height

$\lvert M\rvert > 1$	$\Longrightarrow$	Image is bigger than object (magnified).
$\lvert M\rvert = 1$	$\Longrightarrow$	Image is unmagnified (like a plane mirror).
$\lvert M\rvert < 1$	$\Longrightarrow$	Image is smaller than object (demagnified).
$M > 0$	$\Longrightarrow$	Image is erect.
$M < 0$	$\Longrightarrow$	Image is inverted.
$M = 0$	$\Longrightarrow$	No image is formed.

❖ 5. Ray Tracing Rules:



• Draw Two Rays From Object to Point on Mirror.

• Draw Perpendicular Line to Mirror From Each Point.

• Use θi=θr To Draw Reflected Rays to Eye.

• Extend Reflected Rays behind Mirror to Find Virtual Image

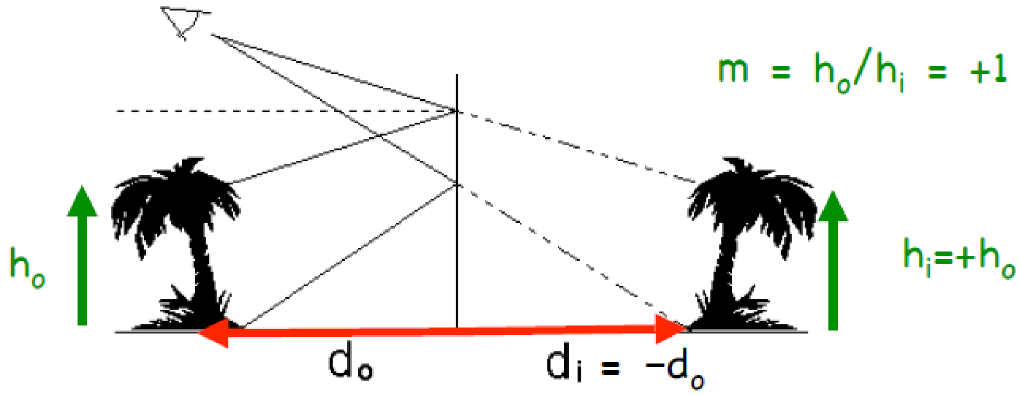
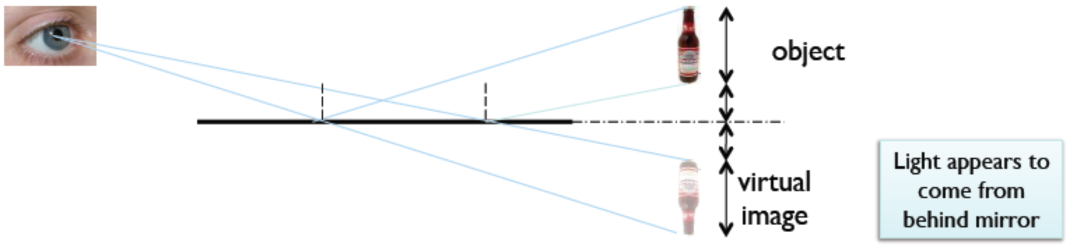


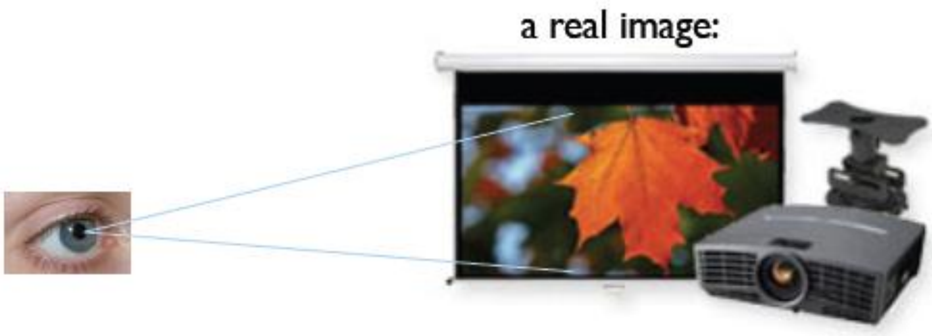
Image distance is minus Object distance, do = –di , means image is behind (minus) the mirror and | do |= |di| means image is behind mirror same distance object is in front.

What is meant by :

Virtual image***:*** means no light actually is at the image location (optical illusion).in other word is meant by the light appears to come from the virtual image, but in fact does not come from there.



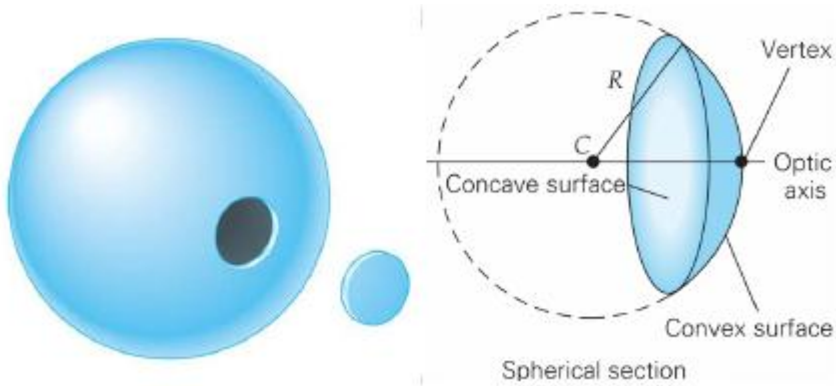
Real image:The light comes from the image (rather than appearing to come from there). You may need a screen to see it.



Magnification m = +1 means image has same height as object (|m|=1) and image is right-side up (m is plus).

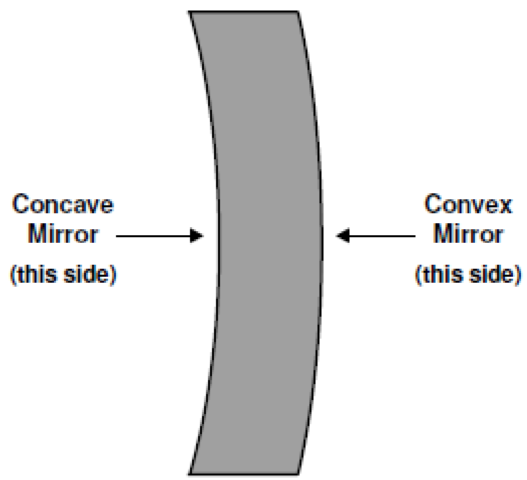
3-5-2 Spherical Mirrors

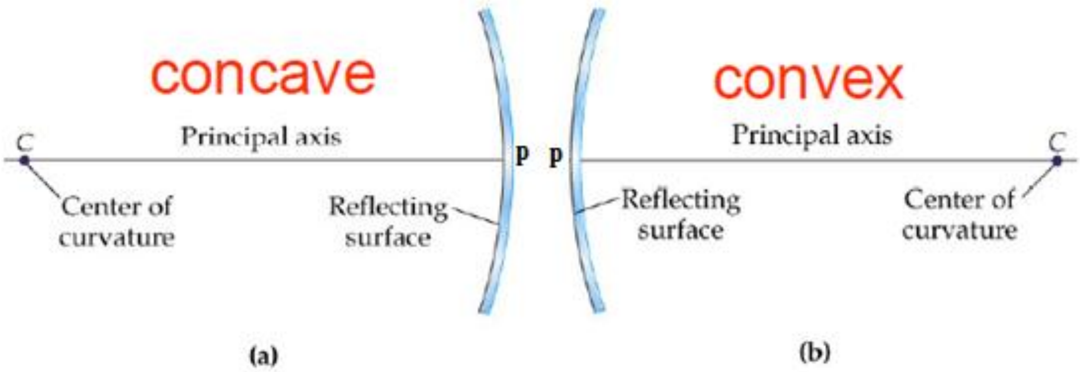
A spherical mirror is a mirror whose surface shape is spherical with radius of curvature R.or in other word is a section of a sphere. It may be concave **[reflective on either the inside]** or convex **[reflective on either the outside]**. **The principal axis (optical axis, vertex) is the straight line between C and the midpoint of the mirror



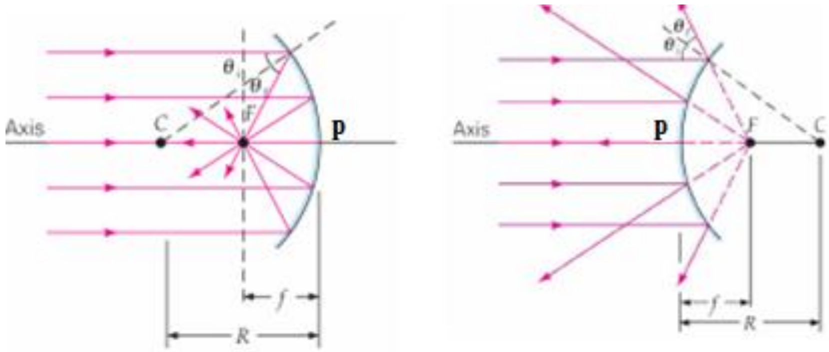
Concave mirror: Reflecting surface is on the “inside” of the curved surface.

Convex mirror: Reflecting surface is on the “outside "of the curved





- **A concave mirror**((*converging*)**(left) focuses incoming parallel rays at the focal point.**
- **A convex mirror** (*diverging*) **bends incoming parallel rays outward (نحو الخارج), as though they came from a focal point behind the mirror**.



Mirrors Images: formed by spherical mirrors may be found by using the parallel, chief, and focal rays.

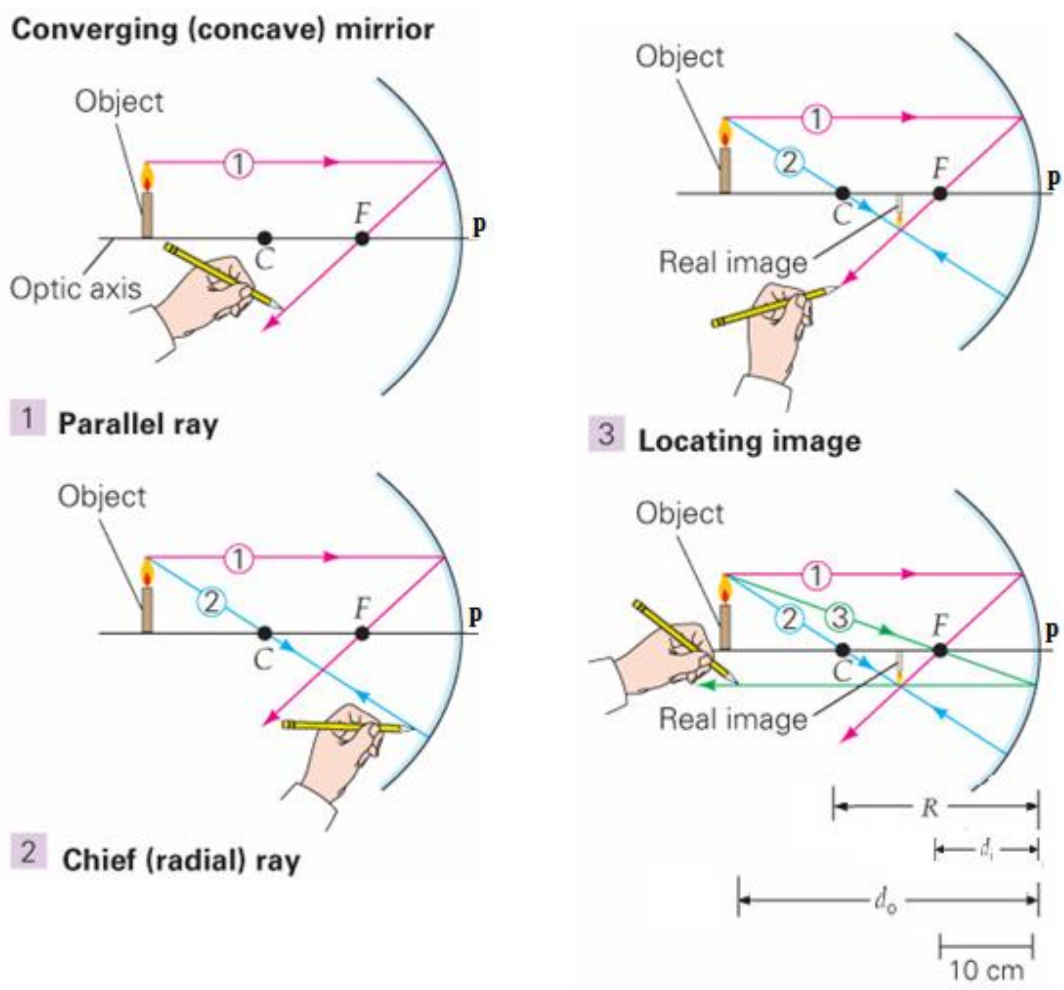
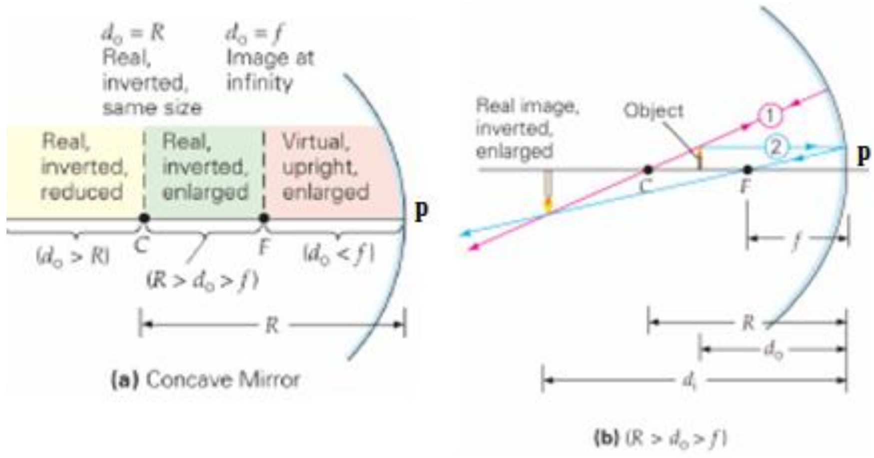


Fig. (3-4): Spherical mirrors

a. A ray of light which is parallel and close to the principal axis of a concave mirror ***F***, of the mirror.[see above fig.1] in other word the incident ray (1) parallel to the principal axis is reflected and then ; after reflection itpasses through the focal point ***F***;
b. The incident ray (*2*) passing through (or travelling towards) ***C*** is reflected back along its original path.
c. The incident ray (*3*) passing through (or travelling towards) ***F*** is reflected back parallel to the principal axis;

For a concave mirror, the type of image formed depends on the position of the object.



(a) Concave Mirror

(b) $(R > d_o > f)$

i. ***C*** is the ***centre of curvature*** of the mirror; it is the centre of the sphere of which the mirror's surface forms part.

ii. ***P*** is the ***pole*** (القطب) of the mirror.

iii. ***CP*** is the ***radius of curvature***, ***R***, of the mirror.

iv. The line through CP is the ***principal axis*** of the mirror.

v. In each case FP is the ***focal length, f*** of the mirror, and:

$$f = R/2$$

vi. **The Mirror Formula**

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = -\frac{R}{2}$$

vii. Where s = object distance, s'= image distance, f = focal length, and R = radius of curvature ***(All distances are measured to the pole of the mirror)***.

In order to distinguish between real and virtual images, and the two types of spherical mirror, it is necessary to employ a sign convention(اتفاقية).There will uses the ***Real is Positive*** convention, in which:

i. The focal lengths and radii of curvature of ***concave*** mirrors are ***<u>positive</u>***, and those of ***convex*** mirrors are ***<u>negative</u>***;

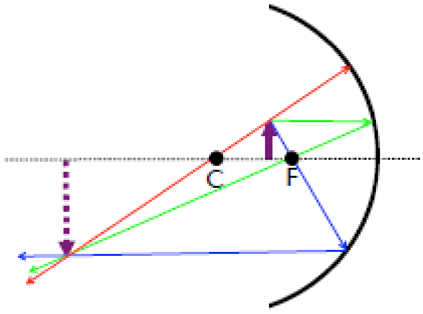
ii. Distances from mirrors to ***real objects*** and ***real images*** are ***<u>positive</u>***, whereas distances to ***virtual objects*** and ***virtual images*** are ***<u>negative</u>***.

viii We can also find the magnification (ratio of image height to object height).

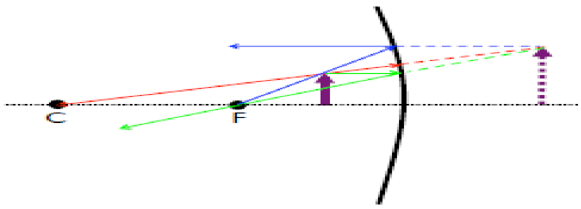
$$m = \frac{hi}{ho} = -\frac{s'}{s} = \frac{f}{f-s}$$

The negative sign indicates that the image is inverted. When s,s' >0 , m<0 inverted and if s/s'<0, m>0 upright or erect (منتصب)

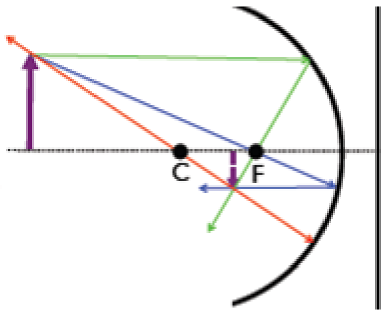
- If the object is between the center of curvature and the focal point, and its image is larger, inverted, and real.



- If an object between focus F and mirror of a concave mirror, its image will be virtual, right-side, further from the mirror than the object; and larger than the object→magnification.

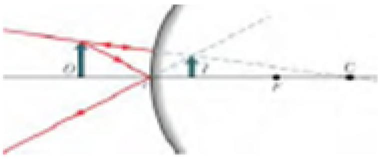


- If an object is outside the center of curvature of a concave mirror, its image will be inverted, smaller, upside down; and real.



Examples:-

1- An object 2cm high is located 10cm from a convex mirror with a radius of curvature of 10cm. Locate(حدد) the image, and find its height.



Focal length: f= r/2= -10cm/2= -5cm.

Image position: 1/s' =1/f -1/s = -1/5cm –1/10cm= -3/10cm

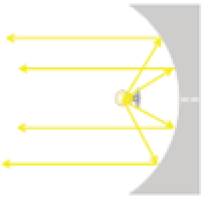
s'= - 10/3cm = -3.33 cm: the image is virtual.

Magnification: m= -s'/s= - (-3.33cm)/(10cm)=+0.33 (upright, smaller).

If the object image is 2cm, the image height is 0.33 x 2cm=0.67 cm.

2- What if we put a light source at the focal point of a concave mirror?

All the rays emitted go through the focal point, and are therefore reflected parallel to the axis of the mirror -> flashlight



3- A convex mirror whose radius of curvature is 30 cm forms an image of a real object which has been placed 20 cm from the mirror. Calculate the position of the image and the magnification produced.

s = + 20 cm (real object), s'= s', R = -30 cm (convex mirror)

$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$$

$$\therefore \quad \frac{1}{20} + \frac{1}{s'} = \frac{2}{-30}$$

$$\text{i.e.} \quad \frac{1}{s'} = \frac{-2}{30} - \frac{1}{20} = \frac{-4}{60} - \frac{3}{60} = \frac{-7}{60}$$

$$s' = \frac{-60}{7} = -8.6cm$$

Since the image distance has turned out to be ***negative***, the image is ***virtual***. The image is therefore 8.6 cm ***behind*** the mirror. The magnification, ***m***, can given by:

$$m = \frac{s'}{s} = \frac{60/7}{20} = \frac{3}{7}$$

Note: The minus sign in ***s'= -60/7*** cm has been ignored. Thus, the magnification is 3/7, i.e. the image is three-sevenths the size of the object.

4- The inside of a spoon bowl is a concave surface with a radius of curvature of a couple of inches. If you hold it about a foot from your face, what will your face look like?

a) Normal size, upside down b) Normal size, right side up

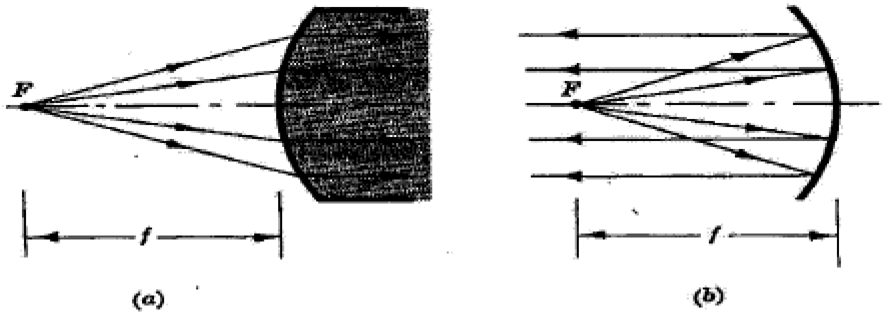
✓ (c) Smaller, upside down d) Smaller, right side up

3-6 Focal points and focal lengths

The ***first focal point*** of a refracting or reflecting surfce, designated ***F***, is: "***axial object point which is imaged by the surface at infinity***".

Rays diverging from the first focal point are parallel to the axis of the surface after reflection or refraction.

Below fig.(a) shows the first focal point F of a refracting surface and fig. (b) that of a concave mirror. The distance from the first focal point to the vertex(قمة الراس) of the surface is called the first focal length, f,



$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R}$$

Fig. (a) First focal point of a refracting surface,
(b) first focal point of a concave mirror.

The first focal length can be computed from Eq.
by setting $s' = \infty$. This gives:

$$\frac{n}{s} + \frac{n'}{\infty} = \frac{n' - n}{R}$$

and since in this case, the object distance s equals the first focal length f,$(\frac{1}{s} + \frac{1}{s\prime} = \frac{1}{f})$

$$f = \frac{n}{n' - n}R$$

For a mirror, we set $n = -n'$, and obtain:

$$f = -\frac{R}{2}$$

If the mirror is concave, as in above fig. (b), R is a negative quantity, f is positive, and the first focal point lies at the left of the vertex, half way between it and the center of curvature.

The ***second focal point*** of a surface designated ***F'***, is:*"the image of an infinitely distant object point on the axis".*

That is, it is the point at which incident rays originally parallel to the axis intersect after reflection or refraction, see below fig. (a).

The distance from the vertex to the second focal point is the ***second focal length, f***.

If we set $s = \infty$ in Eq.

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{R}$$

we find:

$$\frac{n}{\infty}+\frac{n'}{s'}=\frac{n'-n}{R}$$

And since $s' = f'$,$(\frac{1}{s}+\frac{1}{s'}=\frac{1}{f'})$

$$\frac{1}{f'}=\frac{n'}{n'-n}R$$

Setting $n' = -n$, we find for the second focal length of a mirror,

$$\frac{1}{f'}=\frac{R}{2}$$

If the mirror is concave, R is negative, s' and f are negative also and, since they refer to *image distances*, the second focal point lies at a distance $R/2$ to the left of the vertex as in above fig.(b).

In other words, *the first and second focal lengths of a mirror are equal, and the first and second focal points of a mirror coincide*.

It is necessary to distinguish between them and ordinarily one simply speaks of the focal point of a mirror. **The first focal length of a mirror is considered to be the first focal length, $f = -R/2$. While the focal lengths of a refracting surface, are unequal, as in above eqs.**

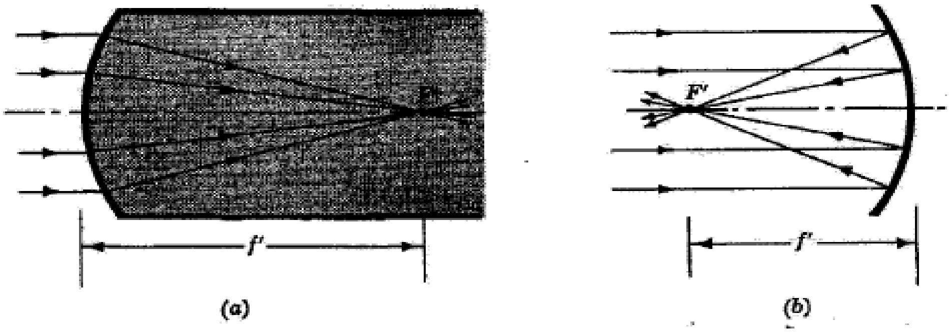
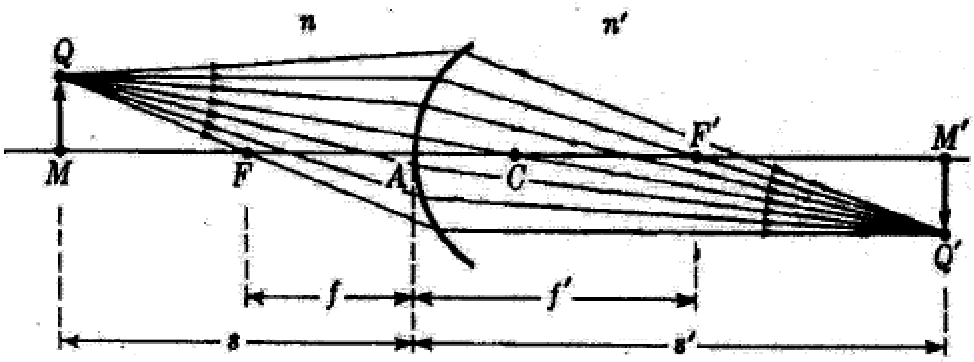


Fig.: (a) Second focal point of a refracting surface, (b) second focal point of a concave mirror.

Virtual Images

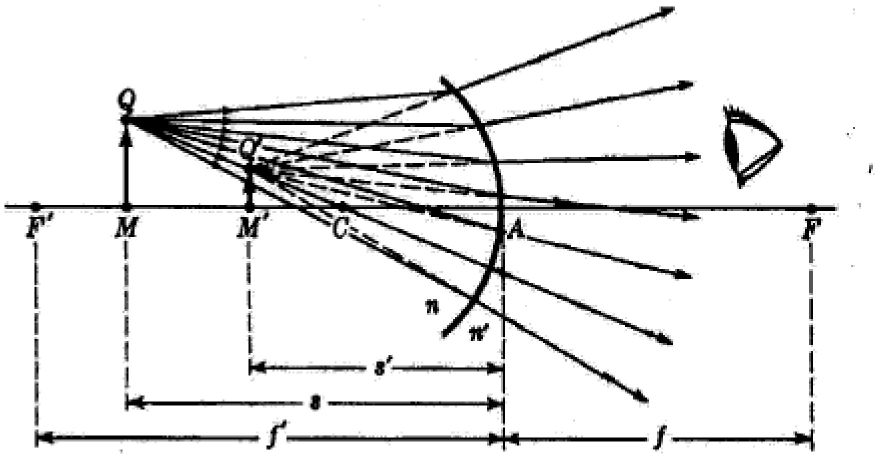
When the eye receives the reflected rays they appear to come from a source, but do not actually pass through. In order to produce a virtual (خيالية) image a surface other than a plane one is required.

The image *M'Q'* in below fig. is a real image in the dense that if a flat screen is located at *M'* a sharply defined image of the object *MQ* will be formed on the screen.



All rays leaving the object point Q, are brought to a focus at the image point Q'.

Not all images can be formed on a screen, as is illustrated in below fig. Light rays from an object point *Q* are shown refracted by a concave spherical surface separating the two media of index ***n=1*** and ***n'=1.5***, respectively. The focal lengths have the ratio ***1:1.5***.



All rays leaving the object point Q, appear to be coming from the virtual image point Q'.

Since the refracted rays are diverging, they will not come to a focus at any point. To an observer's eye located at the right, such rays will appear to be coming from the common point *Q'*. In other words, *Q'* is the image point corresponding to the object point *Q*. similarly M' is the image point corresponding to the object point M. Since the refracted rays do not come from *Q'* but only appear to do so, no image can be formed on a screen placed at M'. For this reason such an image is said to be ***virtual***.

University of Bagdad
College of Science
Department of Physics
Dr.Sarmed S. Alawadi

Subject: Optics I
Semester: first
Year: First year

Chapter Four

Lenses

4-1 Lenses Terminology (مصطلحات)

A lens is an optical system bounded by two or more refracting surfaces having a common axis. If the lens has two surfaces only, it is called a ***simple lens***; if there are more than two surfaces, a ***compound lens***. All high quality lenses are compound lenses, that is, they consist of a number of simple lenses having a common axis. The surfaces of the lenses may be in contact, or there may be air spaces between them.

Except when it is incident normally on one of the surfaces, a ray passing through a simple lens undergoes deviation at both surfaces. The axial thickness of many simple lenses is sufficiently small so that the entire deviation of a ray can be considered to take place in a single plane through the center of the lens. When this approximation can be made, the lens is called a ***thin lens***. Any lens, simple or compound, that is not a thin lens, is called for brevity(باختصار)a ***thick lens***.

Most of lenses and mirrors are spherical or plane, because only spherical and plane surfaces can be produced by machine methods at reasonable cost. When a train of light waves passes through an optical instrument, the curvature of the wave fronts is altered(يتغير)at each boundary surface.

A ray, in its passage through an optical instrument, is made up of a number of segments(تقاطع)of straight lines, deviated at reflecting or refracting surfaces by angles which can be computed from the law of reflection or from Snell's law. The problem of tracing(تتبع)the path of a ray reduces to a problem in geometry and this branch of optics is called ***geometrical optics***.

4-2 Thin Lenses

***Lenses* :are refractive optical devices with two spherical sides.** Diagrams of several standard forms of thin lenses are shown in fig. (4-1). They are shown there as illustrations of the fact that most lenses have surfaces that are spherical in form. Some surfaces are convex, others are concave, and still others are plane. When light passes through any lens, refraction at each of its surfaces contributes to its image forming properties. Not only does each individual surface have its own primary and secondary focal points and planes, but the lens as a whole has its own(خاص)pair of focal points and focal planes. In other word the most common use optical component is the lens with either one on two curved surfaces. Lenses are classified into six types,

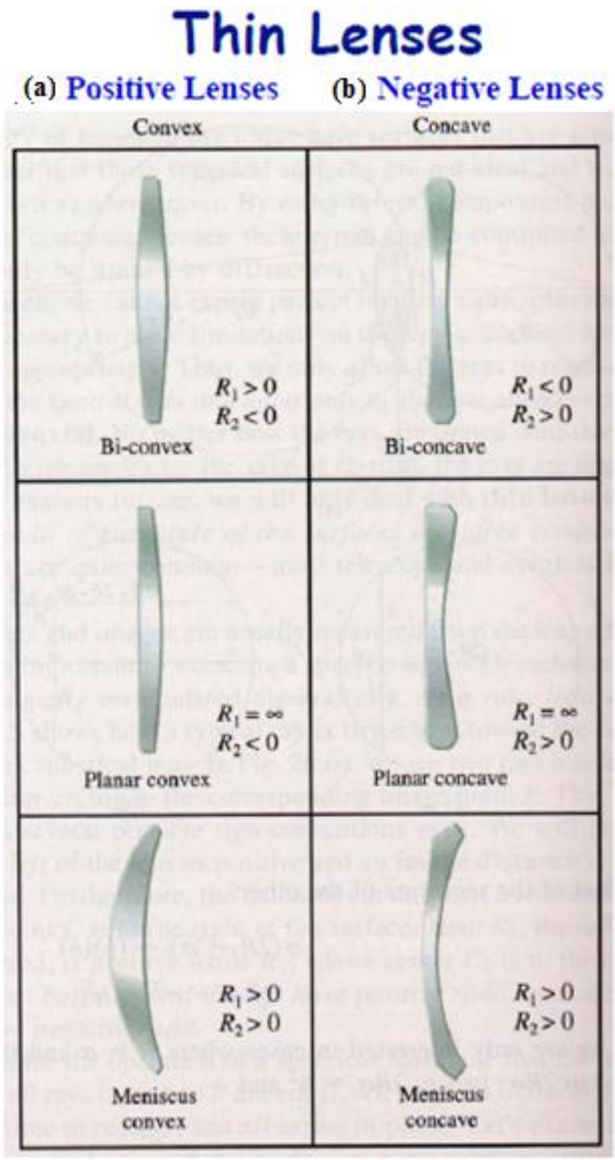


Fig. (4-1): (a) Types of converging lenses, (b) types of diverging lenses

A thin lens may be defined as one whose thickness is considered small in comparison with the distances generally associated with its optical properties and are either *Positive* or *Negative* depending on their *focal length*. Such distances are, for example, radii of curvature of the two spherical surfaces, primary and secondary focal lengths, and object and image distances.

Lenses are of two kinds, converging and diverging. A converging lens is thicker in the middle than at its rim; a diverging lens thinner in the middle.

As in fig. (4-2a), a converging lens brings parallel beam of light to a single focal point F. Here *F* is called a ***real focal point*** because the light rays pass through it.

A diverging lens spreads out a parallel beam of tight so that the rays seem to have come from a focal point *F* behind the lens, as in fig. (4-2b). In this case F is called a ***virtual focal point*** because the tight rays do not actually pass through it but only appear to.

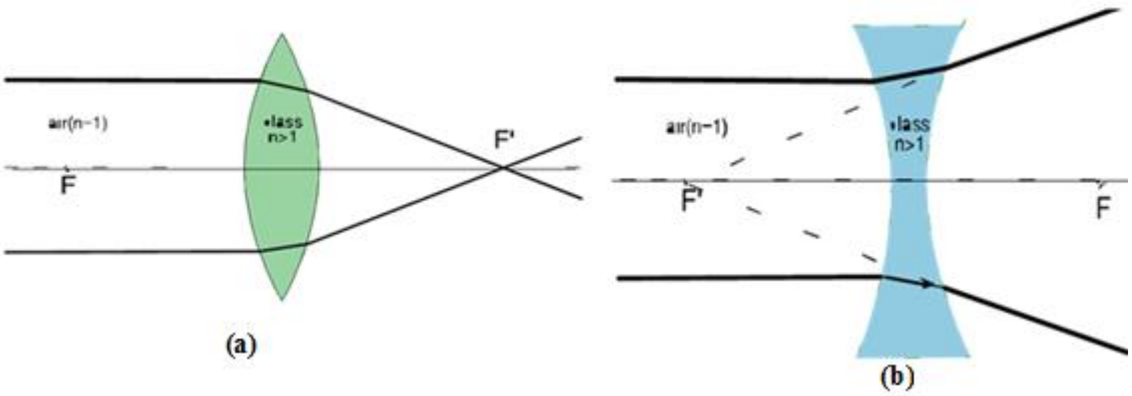


Fig. (4-2): (a) Converging lens, (b) Diverging lens.

Example 1:-

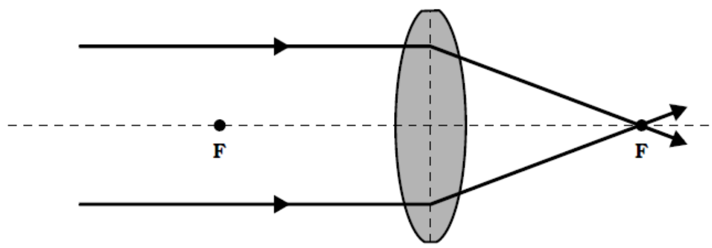
Compare between **Converging lens, and Diverging lens**

Solution:

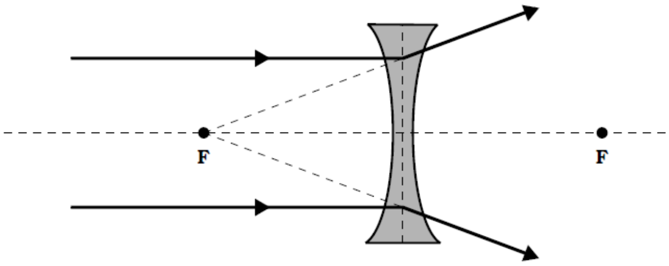
a) Converging lens:

i) Lens thicker at center than edges.

ii) Light rays are refracted towards the focal point, F, on the other side of the lens.



b) Diverging lens:
i) Lens thinner at center than edges.
ii) Light rays are refracted in a direction away fromthe focal point, F, on the inner side of the lens.



4-3 Focal Points and Focal Lengths

Diagrams showing the refraction of light by an equiconvex lens and by an equiconcave lens are given in fig. (4-3).The axis in each case is a straight line through the geometrical center of the lens and perpendicular to the two faces at the points of intersection. For spherical lenses this line joins the centers of curvature of the two surfaces.

تعريفFocal Point (F): **the point to which light beam parallel to principal axis converge.**

The <u>primary focal point F</u> is an axial point having the property that any ray coming from it, or proceeding toward it, travels parallel to the axis after refraction .

Every thin lens in air has two focal points, one on each side of the lens and equidistant from the center).

The <u>secondary focal point F'</u> is an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, proceed toward, or appear to come from, F'.

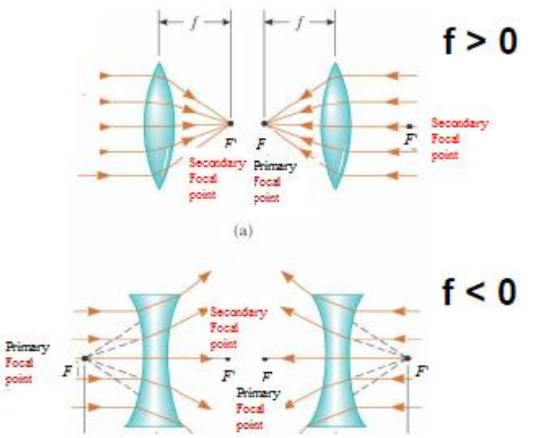


Fig. (4-3): Primary and secondary focal point ; F and F'.

مهمThe distance between the center of a lens and either of its focal points is called its focal length. These distances, designated f and f', usually measured in centimeters or inches, have a positive sign for converging lenses and a negative sign for diverging lenses. In other word **Focal Length (f): Distance between focal point and the mirror or lens.**

It should be noted in fig. (4-3) that the primary focal point F for a converging lens lies to the left of the lens, whereas for a diverging lens it lies to the right. For a lens with the same medium on both sides, we have, by the reversibility of the light rays, $f = f'$,*and* F is the focal point. f is the focal length, an important characteristic

A plane perpendicular to the axis and passing through a focal point is i.e. the significance of the focal plane is illustrated for a converging lens in fig. (4-4).

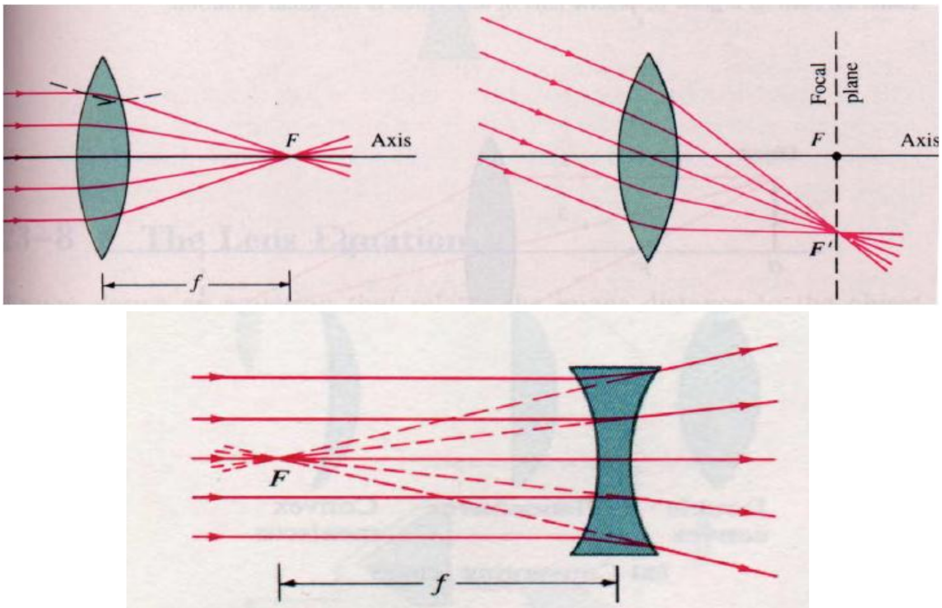
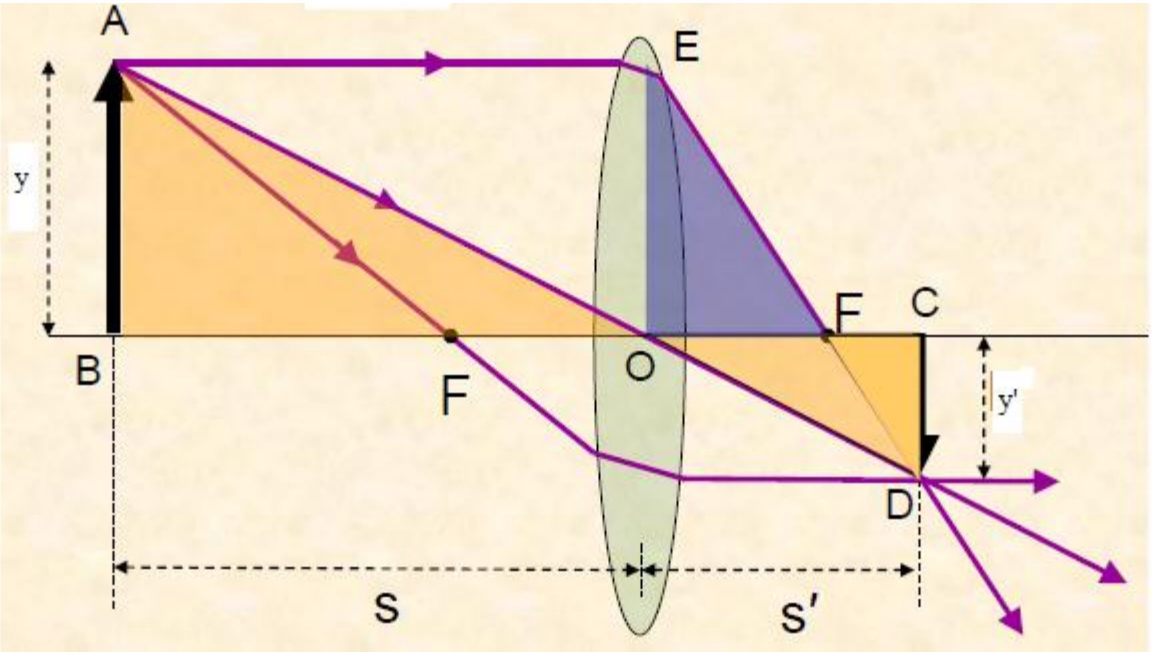


Fig. (4-4): Paraller incident rays are brought to a focus at the secondary focal plane of a thin lens.

4-5 Image Formation--- Gaussian formula of thin lenses calculation

If we know the focal length of a thin lens and the position of an object, there are three methods of determining the position of the image. One is by graphical construction, the second is by experiment, and the third is by use of the lens formula.

So we can drive the lens formula by Ray diagrams which are useful in sketching the relationship between object and image. Relationship may also be calculated



Triangles AOB and DOC are similar

$$\frac{y}{s} = \frac{y'}{s'}$$

$$\frac{y'}{y} = -\frac{s'}{s}$$

Triangles EFO and DCF are similar

$$\frac{y}{f} = -\frac{y'}{s' - f}$$

$$\frac{y'}{y} = -\frac{s' - f}{f}$$

$$\frac{s'}{s} = -\frac{s' - f}{f}$$

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \qquad (4-1)$$

Object distance s

□positive if object is in front of lens

□negative if object is behind lens

Image distance s′

□positive if image is formed behind the lens (real)

□negative if is formed in of the lens (virtual)

Focal length f

•positive -- convex lens

•negative --concave lens

(تعريف القدرة)The reciprocal focal length, called lens *power* or (Strength) $P=\frac{1}{f}$ (f is in meter) The unit of lens power is the *Diopter*, with conventional label D, with units m^{-1}. Most often used in relation of visual system, power of spectacle lenses always quoted in Diopters.

Power	Focal Length
5 D	200 mm
10 D	100 mm
-5 D	-200 mm
-10 D	-100 mm

Converging lenses have a positive power, while diverging lenses have a negative power. By use of the lens makers' formula; eq. (4-2) we may write:

$$p = (n-1)\left(\frac{1}{R1} + \frac{1}{R2}\right)$$

where R_1 and R_2 are the two radii, measured in meters, and n is the refractive index of the glass.

Magnification is defined as

$$M = -\frac{s'}{s} = \frac{y'}{y}$$

M Negative :

□ inverted image

M Positive:

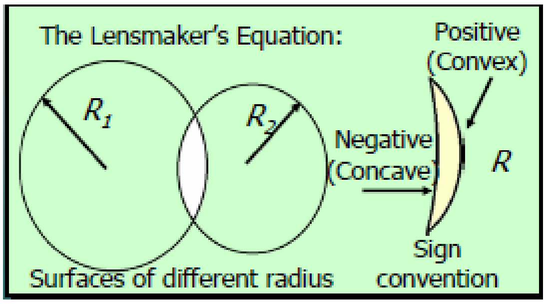
□ Upright image

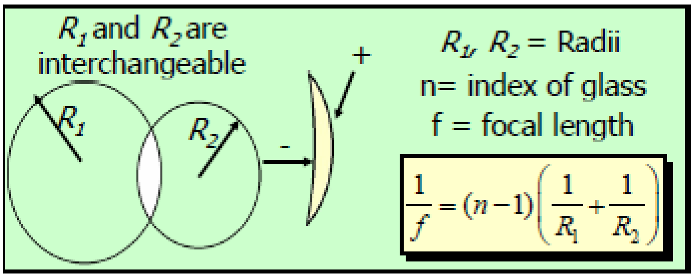
4-8 Lensmaker's Equation

If a lens is to be ground to some specified focal length, the refractive index of the glass must be known, the radii of curvature must be so chosen as to satisfy the equation:

The focal length f for a lens:- $\frac{1}{f}$ =(n-1)($\frac{1}{R1}+\frac{1}{R2}$) $\quad (4-2)$

<u>Important note:</u>





1. R_1 and R_2 are positive for convex outward surface and negative for concave surface.

2. Focal length f is positive for converging and negative for diverging lenses.

Substituting the value of *l/f* from lens formula, we may write:

$$\frac{1}{s'}+\frac{1}{s}=(n-1)\left(\frac{1}{R1}+\frac{1}{R2}\right) \qquad (4-3)$$

Examples

1- A plano-convex lens having a focal length of 25 cm, fig. (4-l) is to be made of glass of refractive index n: 1.520. Calculate the radius of curvature of the grinding and polishing tools that must be used to make this lens.

Solution:

Since a plano-convex lens has one flat surface, the radius for that surface is in infinite, and r_1 in eq. (4-2) is replaced by ∞. The radius r_2 of the second surface is the unknown. Substitution of the known quantities in eq. (4-2) gives:

$$\frac{1}{25}=(1.52-1)(\frac{1}{\infty}-\frac{1}{r_2})$$

Transposing and solving for r_2;

$$\frac{1}{25} = 0.52(0 - \frac{1}{r_2}) = -\frac{0.52}{r_2}$$

r_2= -(25 X 0.520) = -13 cm

If this lens is turned around, as in the figure, we will have r_1 = +13 cm, $r_2 = \infty$.

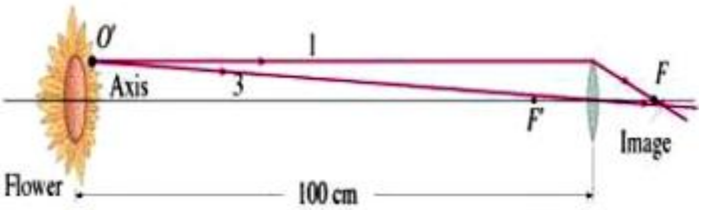
2- The radii of both surfaces of an equiconvex lens of index 1.60 are equal to 8 cm. Find its Power.

Solution:

The given quantities to be used in the above equation are n= 1.60, r_1 = 0.08 m, andr_2 = -0.08 m.

$$P = (n-1)(\frac{1}{r_1} - \frac{1}{r_2}) = (1.6-1)(\frac{1}{0.08} - \frac{1}{-0.08}) = 0.6(\frac{2}{0.08}) = +15D.$$

3- What is (a) the position, and (b) the size, of a large 7.6cm high flower placed 1m from 50mm focal length camera lens?



Solution

بالاستعانة بالشكل اعلاه وباستخدام معادلة العدسات نحصل على الاتي

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{d_i} = \frac{1}{d_o} - \frac{1}{f} = \frac{1}{5cm} - \frac{1}{100cm} = \frac{20-1}{100cm}$$

وعليه

$$d_i = \frac{100cm}{19} = 5.26cm$$

وللحصول حجم الصورة نحسب التكبير على النحو التالي

$$m = -\frac{d_i}{d_o} = -\frac{5.26cm}{100cm} = -0.0526$$

$$h_i = mh_o = -0.0526 \times 7.6cm = -0.4cm$$

أي ان ارتفاع الصورة 4mm ومقلوبة لان اشارة التكبير سالبة

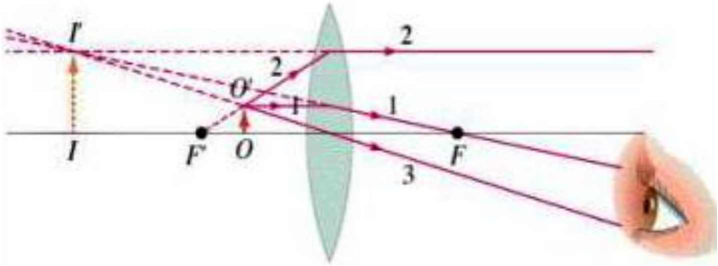
4- An object is placed 10cm from a 15cm focal length converging lens. Determine the image position and size.

Solution

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

$$\frac{1}{d_i}=\frac{1}{d_o}-\frac{1}{f}=\frac{1}{15cm}-\frac{1}{10cm}=-\frac{1}{30cm}$$

$$d_i=-30cm$$



اي ان الصورة خيالية وللحصول حجم الصورة نحسب التكبير على النحو التالي

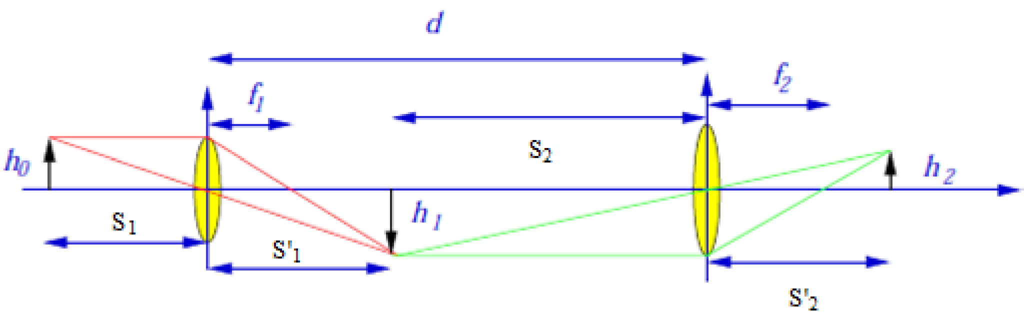
$$m=-\frac{d_i}{d_o}=-\frac{-30cm}{10cm}=3$$

الصورة مكبرة ومعتدلة وتخيلية.

4-9 Compound Lenses and Equivalent Focal Length

For the purpose of minimizing aberrations, most lenses in optical instruments are compound, that is, they consist of several simple lenses having a common axis. The surfaces of adjacent lenses may be in contact, or there may be air spaces between them.

Consider two *well separated* lenses



The simple rule is

The *image* of the first lens becomes the *object* for the next

First lens of focal length f_1 forms an image at S_1 where

$$\frac{1}{s'2} = \frac{1}{f2} - \frac{1}{s2} = \frac{1}{f2} - \frac{1}{d - s'1}$$

$$\frac{1}{s'1} = \frac{1}{f1} - \frac{1}{s1}$$

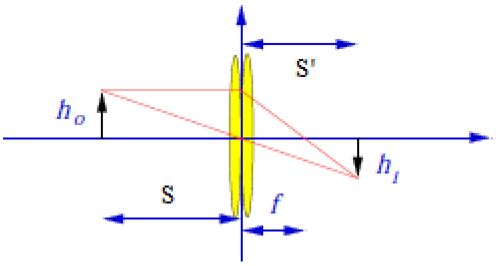
This image then becomes the object for the second lens with object distance

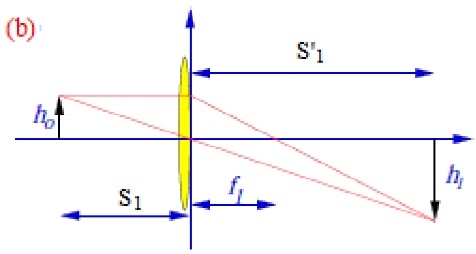
$$S_2 = d - S'_1$$

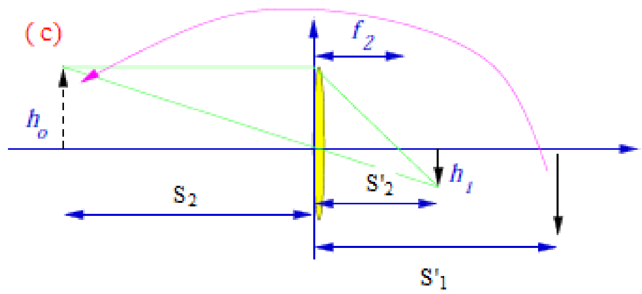
Assumed to be positive, which then form an image as S'_2, where substitute for s'_1 to produce a very messy equation for s'_2 in terms of s_1, f_1, f_2&d.

- For Special case of two thin lenses of focal length $f1$ and $f2$ in contact

(a)







The first lens gives *image* located at S_1

$$\frac{1}{s'1} = \frac{1}{f1} - \frac{1}{s1}$$

The second lens has a virtual object located at

$$S_2 = -S'_1$$

so it will form an image at, S'_2, given by

$$\frac{1}{s'2} = \frac{1}{f2} - \frac{1}{s2} = \left[\frac{1}{f1} + \frac{1}{f2}\right] + \frac{1}{s1}$$

which is just the Gaussian lens formula for a single lens of effective focal length fe, where

$$\frac{1}{fe} = \left[\frac{1}{f1} + \frac{1}{f2}\right]$$

so lenses is contact *add in parallel*.

Effective strength (Seff) of combination of a number of thin lenses close together

$$p_{eff} = p_1 + p_2 + p_3 + \ldots..$$

4-10 Thick Lens Optics

When the thickness of a lens cannot be considered as small compared with its focal length. The lens must be treated as a thick lens. The thick lens may therefore include several component lenses, which may or may not be in contact. Or it is an optical system may contain many lenses, but can be characterized by a few numbers

A simple form of thick lens comprises two spherical surfaces as shown in below fig. . Each surface, acting as an image-forming component, contributes to the final image formed by the system as a whole.

Let n, n', and n'' represent the refractive indices of three media separated by two spherical surfaces of radius r_1 and r_2. A light ray from an axial object point M is shown refracted by the first surface in a direction $T_1M'$and then further refracted by the second surface in a direction T_2M''.

M and M'' are conjugate points for the thick lens as whole, and all rays from M should come to a focus at M''.

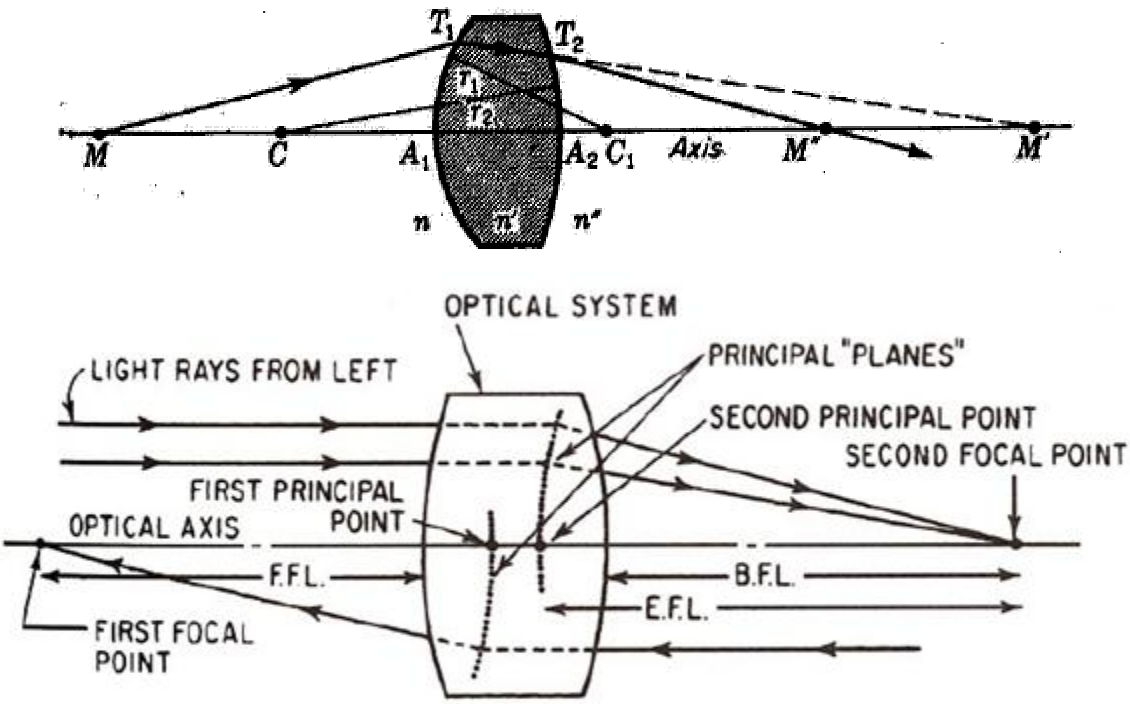


Fig. Refraction of a ray at both surfaces of a thick lens.

The general formulas given for calculating image distances:

$$\frac{n}{s_1}+\frac{n'}{s_1'}=\frac{n'-n}{r_1} \qquad \cdots \textit{for first surface}$$

$$\frac{n'}{s_2'}+\frac{n''}{s_2''}=\frac{n''-n'}{r_2} \qquad \cdots \textit{for second surface}$$

Consider the simple thick lens illustrated in below fig.. The second focal point *F'* of this lens is located by finding the position of the image of an infinitely distant axial object point. That is, we let s_1 in the same fig. equal infinity, find s_1', then, s_2, and finally s_2'. This gives the position of the second focal point, measured from the second vertex.

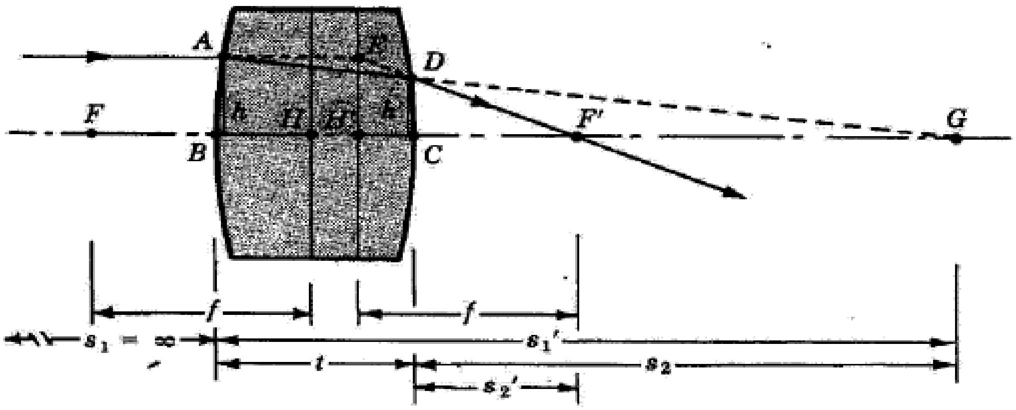


Fig.: A simple thick lens.

The next step is to calculate the focal length of the lens. Above Fig. shows a single ray originating at an infinitely distant axial object point. This ray is incident on the first surface of the lens at point *A*, at a height *h* above the axis, and it leaves the second surface at point D, at a height *h'*.
Projections of the incident and emergent rays intersect at point *E*, which locates the second principal plane and the second principal point, *H'*.
From the similar triangles *ABG* and *DCG*, to within the precision of first order theory,

$$\frac{h}{s_1'}=\frac{h'}{-s_2}$$

From the similar triangles *EH'F'* and *DCF'*,

$$\frac{h}{f} = \frac{h'}{s_2{}'}$$

When the first of these equations is divided the second, we get:

$$\frac{f_1}{s'_1} = -\frac{s'_1}{s_2}$$

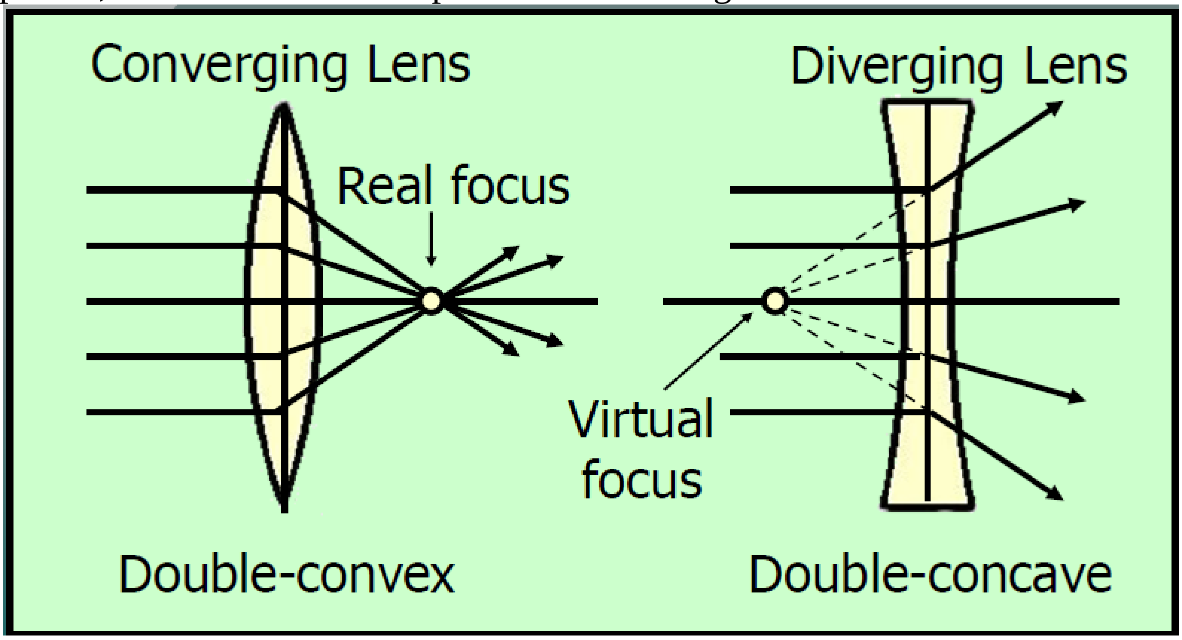
Or

$$f = s'_1\left(-\frac{s'_2}{s_2}\right)$$

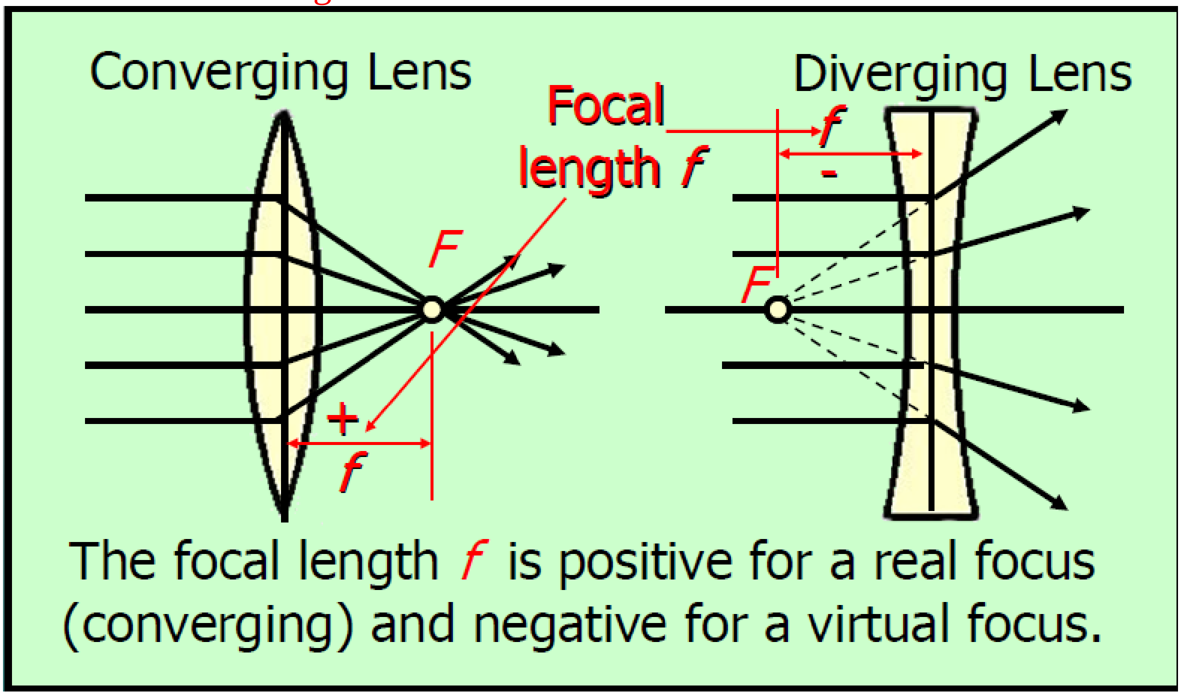
The distances s_1', s_2 must all be computed in the process of locating the second focal point.

Summary:

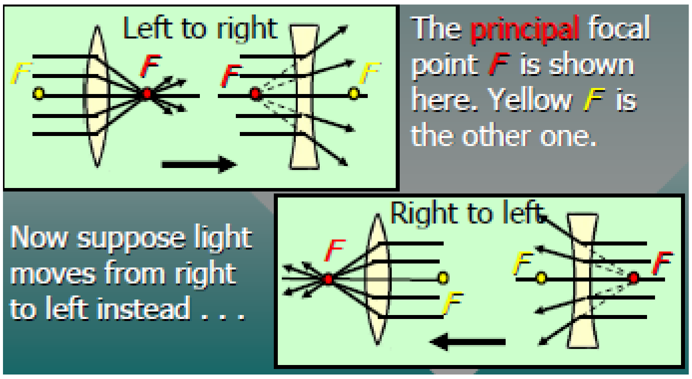
Converging and Diverging Lens If a smooth surface replaces the prisms, a well-defined focus produces clear images.



The Focal Length of Lenses

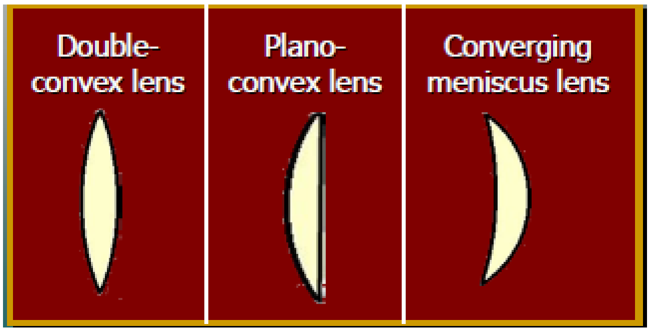


The Principal Focus Since light can pass through a lens



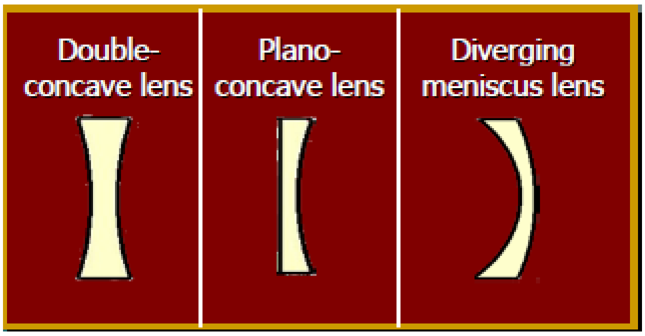
Types of Converging Lenses

In order for a lens to converge light it must be thicker near the midpoint to allow more bending.



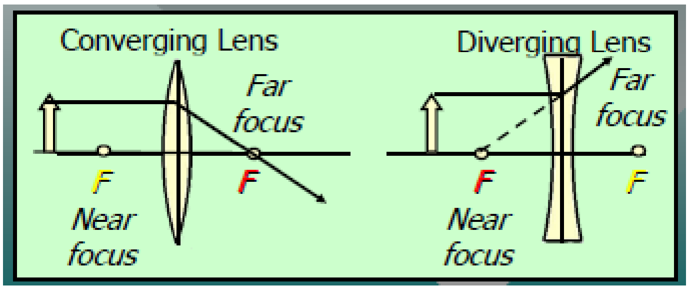
Types of Diverging Lenses

In order for a lens to diverge light, it must be thinner near the midpoint to allow more bending.



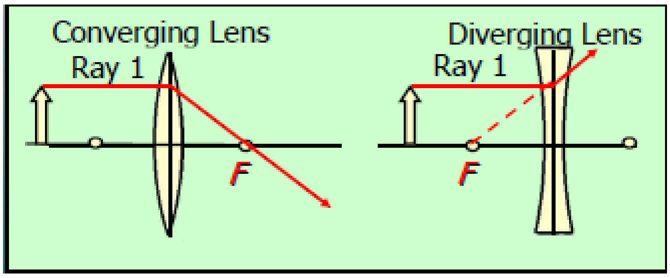
Terms for Image Construction

1- The near focal point is the focus *F* on the same side of the lens as the incident light.
2- The far focal point is the focus F on the opposite side to the incident light.

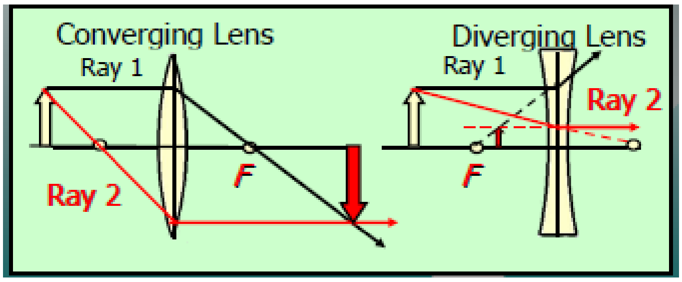


- Image Construction(تكون):

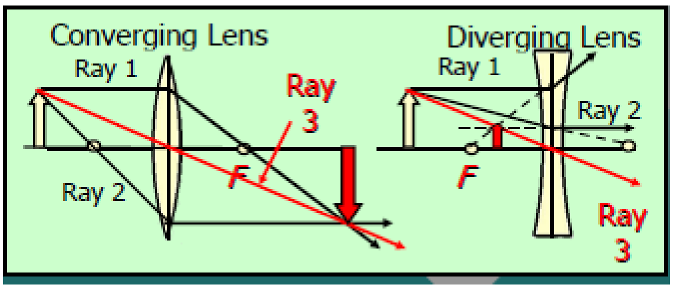
Ray 1: A ray parallel to the lens axis passes through the far focus of a converging lens or appears to come from the near focus of a diverging lens.



Ray 2: A ray passing through the near focal point of a converging lens or proceeding toward the far focal point of a diverging lens is refracted parallel to the lens



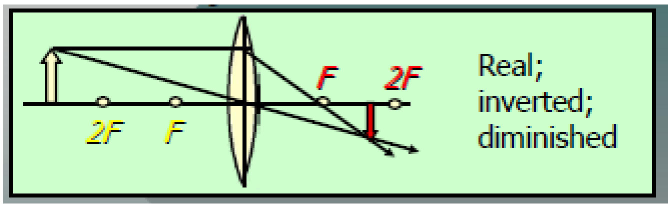
Ray 3: A ray passing through the center of any lens continues in a straight line. The refraction at the first surface is balanced by the refraction at the second surface.



ملاحظات مهمة:-

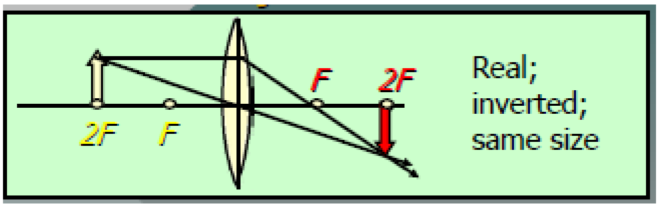
1- Object Outside 2F

- The image is inverted; i.e., opposite to the object orientation.
- **The image is real i.e., formed by actual light rays in front of** mirror.
- The image is diminished in size; i.e., smaller than the object.
- Image is located between F and 2F



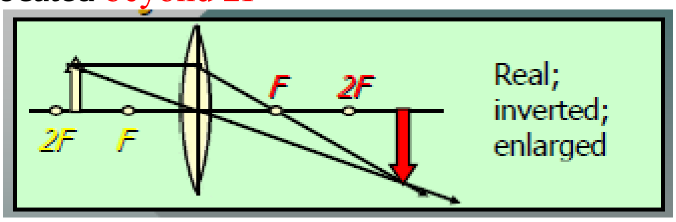
2- Object at 2F

- The image is inverted ; i.e., opposite to the object orientation.
- The image is real ; i.e., formed by actual light rays in front of the mirror.
- The image is the same size as the object.
- Image is located at 2F on other side.



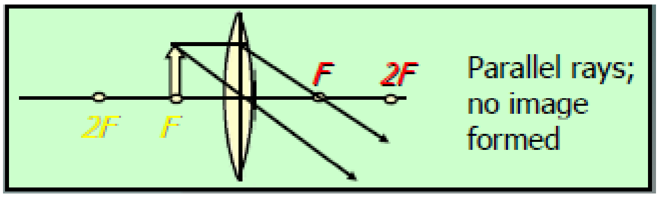
3- Object Between 2F and F

- The image is inverted; i.e., opposite to the object orientation.
- The image is real ; formed by actual light rays on the opposite side
- The image is enlarged in size; i.e., larger than the object.
- Image iFs located beyond 2F



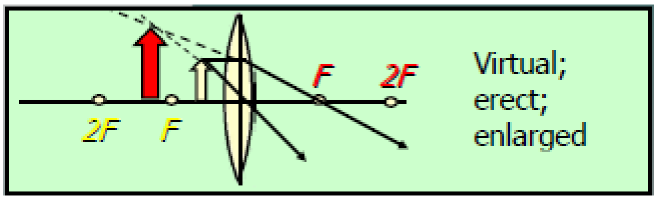
4- Object at Focal Length F

- When the object is located at the focal length, the rays of light are parallel. The lines never cross, and no image is formed.



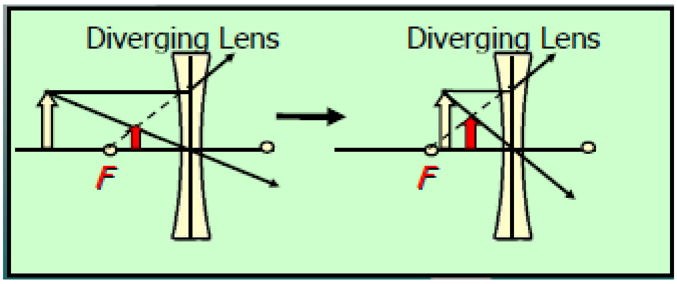
5- Object Inside F

- The image is erect; i.e., same orientation as the object.
- The image is virtual; i.e., formed where light does NOT go.
- The image is enlarged in size; i.e., larger than the object.
- Image is located on near side of lens



Diverging Lens Imaging

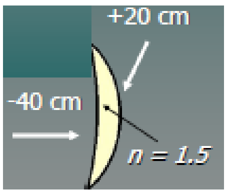
- All images formed by diverging lenses are erect, virtual, and diminished. Images get larger as object approaches.



Examples:

1- A glass meniscus lens (n = 1.5) has a concave surface of radius –40 cm and a convex surface whose radius is +20 cm. What is the focal length of the lens?

$R_1=20$, $R_2 = -40$ cm



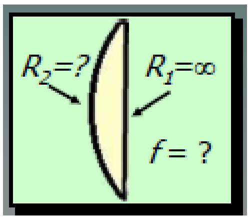
$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20\text{cm}} + \frac{1}{(-40\text{cm})}\right) = (0.5)(\frac{2-1}{40\text{cm}})$$

f=20.0 cm converging (+)lens

2- What must be the radius of the curved surface in a plano -convex lens in order that the focal length be 25 cm?

$R_1 = \infty$, f = 25cm, R_2=??



$R_1 = \infty$, f = 25cm, R_2=??

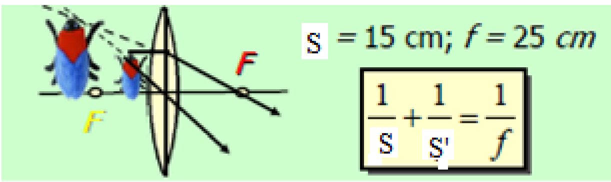
$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$$

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$$

$$\frac{1}{25\ cm} = (1.5-1)(\frac{1}{\infty} + \frac{1}{R_2})$$

R_2 = 0.5(25cm)= 12.5cm convex(+)surface

3- A magnifying glass consists of a converging lens of focal length 25 cm. A bug is 8 mm long and placed 15 cm from the lens. What are the nature, size, and location of the image?

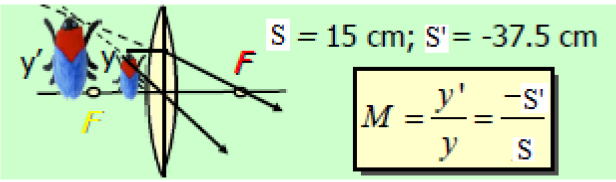


1- The nature and location:-

$$s' = \frac{sf}{s-f} = \frac{(15cm)(25cm)}{(15cm - 25cm)} = -37.5cm$$

The fact that s' is negative means that the image is virtual (on same side as object).

2- The size:- A magnifying glass consists of a converging lens of focal length 25 cm. A bug is 8 mm long and placed 15 cm from the lens. What is the size of the image?

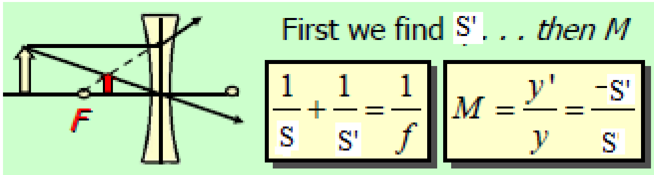


$$M = \frac{y'}{8mm} = \frac{-(-37.5cm)}{15cm}$$

y' = +20mm

The fact that y' is positive means that the image is erect. It is also larger than object.

4- What is the magnification of a diverging lens (f = - 20 cm) if the object is located 35cm from the center of the lens?



$$s' = \frac{sf}{s-f} = \frac{(35cm)(-20cm)}{35cm - (-20cm)} = +12.7cm$$

$$M = \frac{s'}{s} = \frac{-(-12.7cm)}{35cm} = +0.364$$

5- Derive an expression for calculating the magnification of a lens when the object distance and focal length are given.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$s' == \frac{sf}{s-f}$$

$$M = \frac{y'}{y} = \frac{-s'}{s}$$

From last equation:

$$s' = -sM$$

Substituting for s' in second equation gives . . .

$$sM = \frac{sf}{s-f} = \frac{-f}{s-f}$$

Use this expression to verify answer in Example 4.

6- An object 2cm high is placed 4cm from a bi-convex lens with r1=10cm and r2=15cm, and index of refraction n=1.5. Find the position and size of the image.

7- An object 0.5 cm in height is placed 8 cm from a convex lens of focal length 10 cm. Determine the position, magnification, orientation and height of the image.

8- An object is placed 45 cm from a lens of focal length -25 cm. Determine the position, magnification, and orientation of the image.

9- Determine the combined strength of a thin convex lens and a thin concave lens placed close together if their respective focal lengths are 10cm and -20cm.